Open boundary conditions and coupling methods for ocean and atmosphere numerical models

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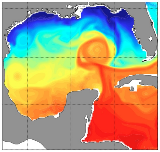
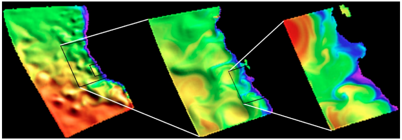
Context

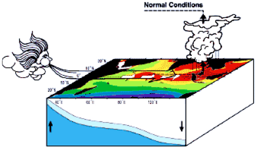
- Limited area models



- Multiscale and/or nested systems



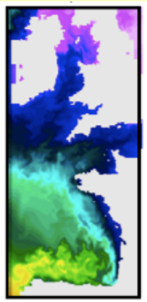
- Coupled systems



$\longrightarrow$ Which interface conditions ?

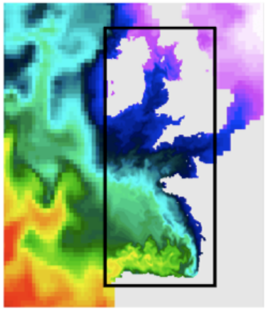
Open boundary problem

Which boundary conditions for regional models ?



Two-way interaction

How can we connect two models in a mathematically correct way ?



Outline

1. The open boundary problem
   - Classification of the methods
   - Numerical experiments in a shallow water model
   - One step further: absorbing boundary conditions

2. Model coupling
   - Formalization and usual methods
   - Schwarz methods

Outline

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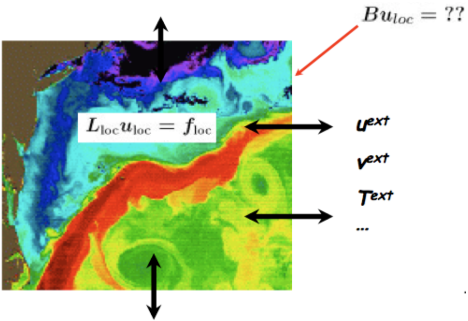
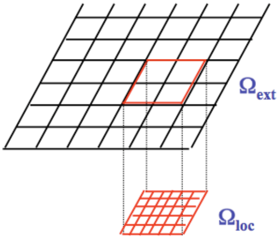
The open boundary problem



A particular case :
one-way nesting



Goal : choose the partial differential operator B in order to

- evacuate the outgoing information
- bring some external knowledge on incoming information

What is done usually

Old problem in ocean-atmosphere modelling : abundant literature, numerous conditions proposed, often with no clear conclusions.

However **a few OBCs are often recommended** in comparative studies : radiation conditions, Flather condition, sponge layer. . .

Interpretation

The performances of usual conditions are fully consistent with the following criterion : $Bw = Bw_{\text{ext}}$ for each incoming characteristic variable w of the hyperbolic part of the equations (Blayo and Debreu, *Ocean Modelling*, 2005).

Usual methods

Radiation conditions

Based on the Sommerfeld condition: $$\frac{\partial \phi}{\partial t} + c\,\frac{\partial \phi}{\partial x} = 0$$

$+$ local adaptive evaluation of c (Orlanski-like methods)

Performances

- OK for simple idealized testcases, where the flow is dominated by a single wave
- Poor for complex flows

Interpretation $w = \dfrac{\partial \phi}{\partial t} + c\,\dfrac{\partial \phi}{\partial x}$ is the incoming characteristic for the wave equation.

Usual methods

Flather condition

For free surface 2-D flows (case of an eastern open boundary) :

Sommerfeld condition for free surface: $\dfrac{\partial h}{\partial t} + \sqrt{gh_0}\,\dfrac{\partial h}{\partial x} = 0$

1-D approximation of the continuity equation: $\dfrac{\partial h}{\partial t} + h_0\,\dfrac{\partial u}{\partial x} = 0$

Combination + integration through Γ: $u - \sqrt{\dfrac{g}{h_0}}\,h = u^{\text{ext}} - \sqrt{\dfrac{g}{h_0}}\,h^{\text{ext}}$

Performances good results in all comparative studies

Interpretation $w_1 = u - \sqrt{\dfrac{g}{h_0}}\,h$ is the incoming characteristic variable of the shallow-water system.

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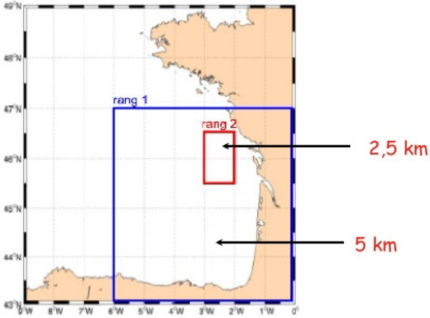
Numerical experiments

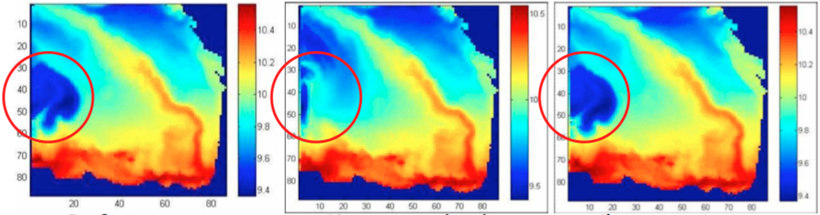
MARS model (IFREMER)
(collaboration: F. Vandermeirsch)



Numerical results

Propagation of a temperature anomaly

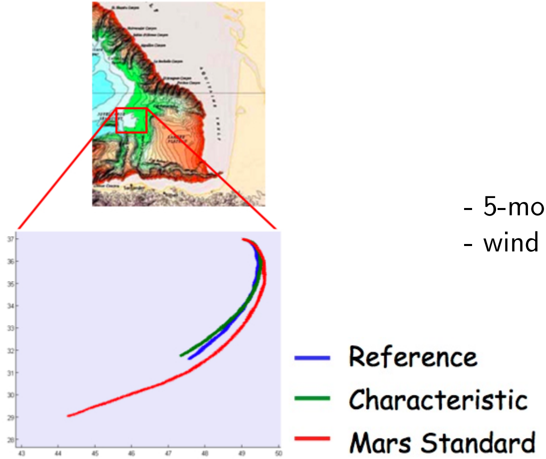
Solution after 2 months



Reference | **Mars standard conditions** | **Characteristic conditions**

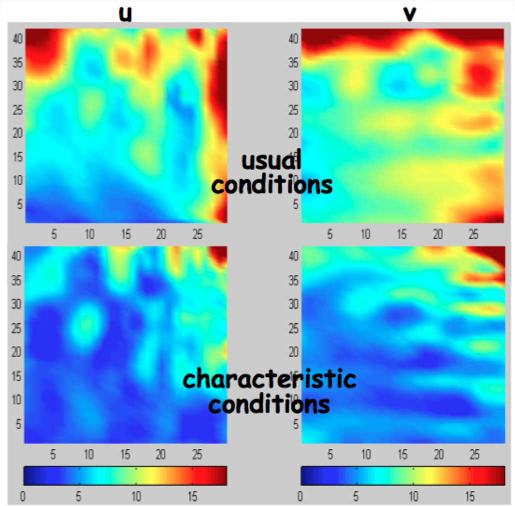
$$\begin{cases} h = h^{\text{ext}} \\ \dfrac{\partial U}{\partial n} = 0 \end{cases}$$

Float trajectories



- 5-month simulation
- wind forcing

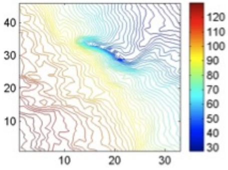
Numerical results



L^2 norm of the error

integrated over 2 months

bathymetry



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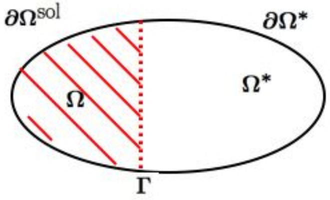
General idea



Reference solution (unknown):

$$\begin{cases} Lu^* = f & \text{in } \Omega^* \times [0,T] \\ Bu^* = g & \text{on } \partial\Omega^* \times [0,T] \\ u^*(t=0) = u_0 & \end{cases}$$

$u^{\rm ext}$: external data (approximation of u^*)

One is looking for u solution of

$$\begin{cases} Lu = f & \text{in } \Omega \times [0,T] \\ Bu = g & \text{on } \partial\Omega^{\rm sol} \times [0,T] \\ Cu = Cu^{\rm ext} & \text{on } \Gamma \times [0,T] \\ u(t=0) = u_0 & \text{in } \Omega \end{cases}$$

$e = u - u^*$ error on u
$e^{\text{ext}} = u^{\text{ext}} - u^*$ error on the data

$$\begin{cases} Le = 0 & \text{in } \Omega \times [0,T] \\ Be = 0 & \text{on } \partial\Omega^{\text{sol}} \times [0,T] \\ Ce = Ce^{\text{ext}} & \text{on } \Gamma \times [0,T] \\ e(t=0) = 0 & \text{in } \Omega \end{cases}$$

$\rightarrow$ If one chooses C such that $Ce^{\text{ext}} = 0$, then $e = 0$ (i.e. $u = u^*$ on Ω)

If one assumes that $Lu^{\text{ext}} \simeq f$, then $Le^{\text{ext}} \simeq 0$.

To be solved:

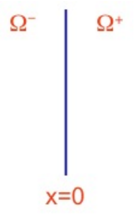
Find C such that $Ce^{\text{ext}} = 0$ on Γ, given that $Le^{\text{ext}} = 0$ on $\Omega^* \setminus \Omega$

$\longrightarrow$ definition of an absorbing condition (Engquist & Majda, 1977)
On our equations: Halpern, 1986; Nataf et al., 1995; Lie, 2001...

Derivation of absorbing conditions

Example: 2-D advection-diffusion-reaction equation

$$Lu = \frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} - \nu\Delta u + cu = f \qquad \text{in } \mathbf{R}^2 \times]0, +\infty[$$



Fourier transform: $\hat{w}(x,k,\omega) = \dfrac{1}{2\pi} \displaystyle\iint w(x,y,t)\, e^{-i(ky+\omega t)}\, dy\, dt$

$$Le = 0 \Longrightarrow \widehat{Le} = -\nu\, \frac{\partial^2 \hat{e}}{\partial x^2} + a\, \frac{\partial \hat{e}}{\partial x} + \left[c + \nu k^2 + i(\omega + bk)\right] \hat{e} = 0$$

Derivation of absorbing conditions

$$\begin{cases} \hat{e}^- = \alpha \ \exp(\lambda^+ x) \\ \hat{e}^+ = \beta \exp(\lambda^- x) \end{cases} \quad \text{with } \lambda^\pm = \frac{1}{2\nu}\left[a \pm \sqrt{a^2 + 4c\nu + 4\nu^2 k^2 + 4i\nu(\omega + bk)}\right]$$

$$\Longrightarrow \begin{cases} \dfrac{\partial \hat{e}^-}{\partial x} - \lambda^+ \hat{e}^- = 0 \Rightarrow \dfrac{\partial e^-}{\partial x} - \Lambda^+ e^- = 0 \\[2ex] \dfrac{\partial \hat{e}^+}{\partial x} - \lambda^- \hat{e}^+ = 0 \Rightarrow \dfrac{\partial e^+}{\partial x} - \Lambda^- e^+ = 0 \end{cases}$$

with $\Lambda^\pm(e) = TF^{-1}(\lambda^\pm \hat{e})$
(Steklov-Poincaré operator)

Ideally: $C = \begin{cases} \dfrac{\partial}{\partial x} - \Lambda^- & \text{if } \Omega = \mathbf{R}^- \\[2ex] \dfrac{\partial}{\partial x} - \Lambda^+ & \text{if } \Omega = \mathbf{R}^+ \end{cases}$

But pseudo-differential operator (non local, both in time and space).

Derivation of absorbing conditions

$\Lambda^{\pm}$ can be approximated by differential operators, at different orders:

$$\lambda_0^{\pm} = \frac{a \pm p}{2\nu} \qquad \text{and} \qquad \lambda_1^{\pm} = \frac{a \pm p}{2\nu} \pm i(\omega + bk)\, q$$

i.e. $$\Lambda_0^{\pm} = \frac{a \pm p}{2\nu} Id \qquad \text{and} \qquad \Lambda_1^{\pm} = \frac{a \pm p}{2\nu} Id \pm q \frac{\partial}{\partial t} \pm bq \frac{\partial}{\partial y}$$

where p and q are coefficients to be determined.

Taylor expansion (assuming k and ω small) :

$$p = \sqrt{a^2 + 4c\nu} \qquad \text{and} \qquad q = 1/\sqrt{a^2 + 4c\nu}$$

Minimization of the reflection ratio $\rho = \dfrac{\text{reflected wave}}{\text{incident wave}}$

0th order: minimize $\rho(p)$ 1st order: minimize $\rho(p, q)$

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Minimization of the reflection ratio $\qquad \rho = \dfrac{\text{reflected wave}}{\text{incident wave}}$

0th order: minimize $\rho(p)$ $\qquad$ 1st order: minimize $\rho(p, q)$

Application to the shallow-water equations

- 0th order (i.e. flat bottom, without friction): $w_1 = 0$
  (we recover a classical method of characteristics)

- 1st order (different possible expansions):
  - flat bottom, weak bottom friction (r) : $\dfrac{\partial w_1}{\partial x} - \dfrac{r}{4c} w_3 = 0$
  - no friction, weak topographic slope (α) :
    $2c\dfrac{\partial w_1}{\partial t} - \alpha u_0 w_1 - \dfrac{\alpha(u_0 + c)}{2} w_3 = 0$
  - no friction, strong topographic slope (minimization of the reflection ratio): $a\dfrac{\partial w_1}{\partial t} + b w_1 - \dfrac{\alpha}{2} w_3 = 0$
    where a, b are solutions of a minmax problem.

Numerical experiments $\longrightarrow$ to be done

with V. Martin (LAMFA Amiens) and F. Vandermeirsch (IFREMER Brest)

Outline

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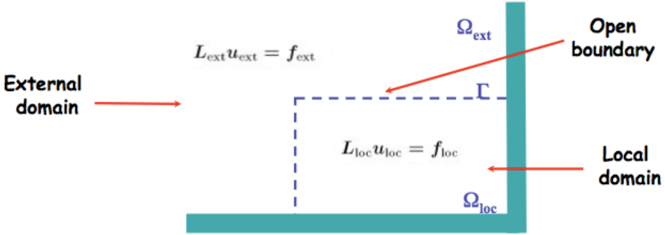
Formalization of the coupling problem

The two models are fully available.



A formulation of the problem could be:

Find u_{ext} and u_{loc} such that

$$\begin{cases} L_{\text{loc}}u_{\text{loc}} = f_{\text{loc}} & \text{in } \Omega_{\text{loc}} \times [0,T] \\ L_{\text{ext}}u_{\text{ext}} = f_{\text{ext}} & \text{in } \Omega_{\text{ext}} \times [0,T] \\ u_{\text{loc}} = u_{\text{ext}} \text{ et } \dfrac{\partial u_{\text{loc}}}{\partial n} = \dfrac{\partial u_{\text{ext}}}{\partial n} & \text{on } \Gamma \times [0,T] \end{cases}$$

However **usual coupling methods** are **ad-hoc simple algorithms** in order to be computationally cheap:

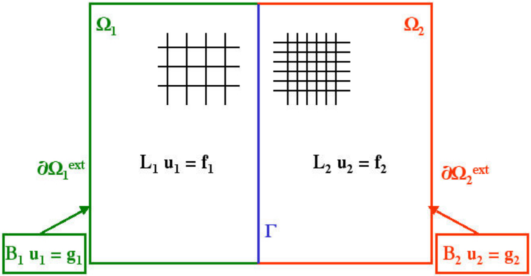
- Run some time steps of the first model
- Send boundary data to the second model
- Run corresponding time steps of the second model
- Send boundary data to the first model
- idem with the next time steps...

$\Rightarrow$ They are **not** fully **satisfactory from a mathematical point of view**.

Question: can we **improve the physical solution** of the coupled system **by improving mathematical aspects** of the coupling method ?

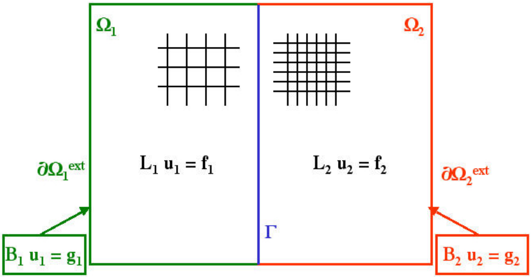
Outline

Framework: Schwarz methods



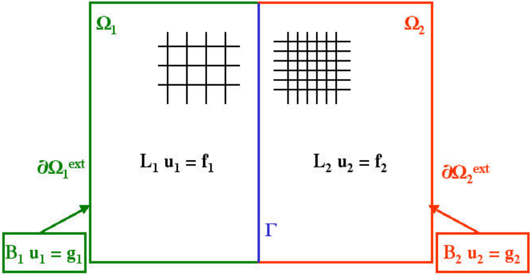
$$\left\{\begin{array}{rll} L_1u_1 & = f_1 & \Omega_1\times[0,T] \\ u_1 & \text{given} & \text{at } t=0 \\ B_1u_1 & = g_1 & \partial\Omega_1^{\text{ext}}\times[0,T] \\ C_1u_1 & = C_1u_2 & \Gamma\times[0,T] \end{array}\right. \qquad \left\{\begin{array}{rll} L_2u_2 & = f_2 & \Omega_2\times[0,T] \\ u_2 & \text{given} & \text{at } t=0 \\ B_2u_2 & = g_2 & \partial\Omega_2^{\text{ext}}\times[0,T] \\ C_2u_2 & = C_2u_1 & \Gamma\times[0,T] \end{array}\right.$$

Framework: Schwarz methods



$$\left\{\begin{array}{lll} L_1 u_1^{n+1} & = f_1 & \Omega_1 \times [0,T] \\ u_1^{n+1} & \text{given} & \text{at } t=0 \\ B_1 u_1^{n+1} & = g_1 & \partial\Omega_1^{\text{ext}} \times [0,T] \\ C_1 u_1^{n+1} & = C_1 u_2^{n} & \Gamma \times [0,T] \end{array}\right. \qquad \left\{\begin{array}{lll} L_2 u_2^{n+1} & = f_2 & \Omega_2 \times [0,T] \\ u_2^{n+1} & \text{given} & \text{at } t=0 \\ B_2 u_2^{n+1} & = g_2 & \partial\Omega_2^{\text{ext}} \times [0,T] \\ C_2 u_2^{n+1} & = C_2 u_1^{n} & \Gamma \times [0,T] \end{array}\right.$$

Framework: Schwarz methods



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Usual coupling methods correspond to one (and only one) iteration, with some particular choice of C_1 and C_2.

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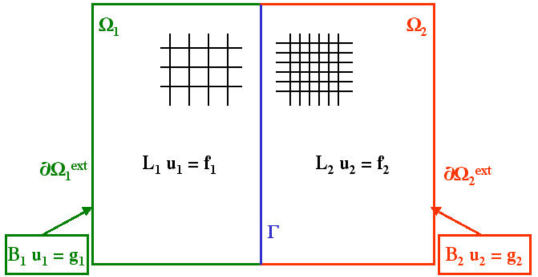

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Usual coupling methods correspond to one (and only one) iteration, with some particular choice of C_1 and C_2.

$\longrightarrow$ Is it worth iterating ? (is there an impact on the physics ?)

$\longrightarrow$ How to reduce the computation cost ?

Impact on the physics

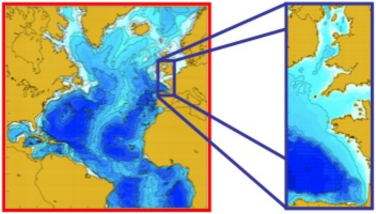
Major difficulty There is **no idealized ocean or atmosphere testcase with a known reference solution**, in the case of the coupling of two different models.

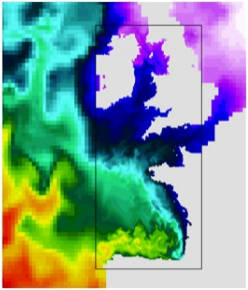
However our numerical experiments make us believe that using a Schwarz iterative method:

- leads to an **improved regularity** of the coupled solution
- seems to **remove a source of error**

Impact on the physics: improved regularity

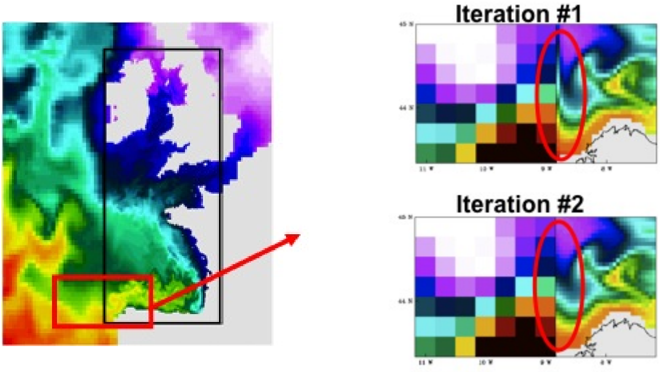
Testcase #1: coupling a $1/3^\circ$ model of the North Atlantic and a $1/15^\circ$ model of the Bay of Biscay (Cailleau et al., *Ocean Modelling*, 2008)





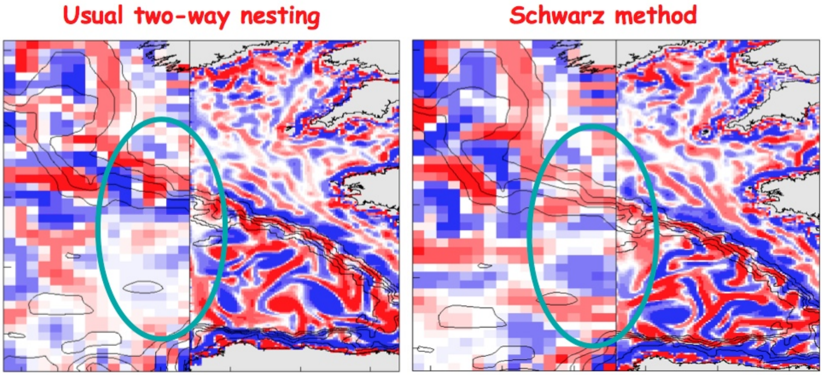
3-year simulation - primitive equation model NEMO

Impact on the physics: improved regularity (2)



Temperature $z = 10m$

Impact on the physics: improved regularity (3)



Instantaneous vorticity field, z=30m

Impact on the physics: more robust solution

Testcase #2: Simulation of the tropical cyclone Erica (2003), by coupling

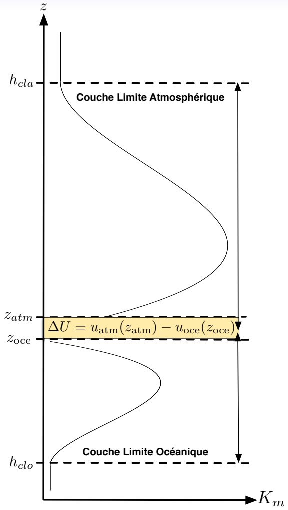
- ROMS: primitive equation ocean model (Shchepetkin-McWilliams, 2005)
- WRF: non hydrostatic atmospheric model (Skamarock-Klemp, 2007)

$\Delta x_a = 35$km, $\Delta t_a = 180$s
$\Delta x_o = 18$km, $\Delta t_o = 1800$s
15-day simulation

Boundary Conditions: vertical fluxes for $\vec{\tau}, Q_{\text{net}}$ and F

$$\rho_a K_z^a \frac{\partial u_{\text{atm}}}{\partial z}(0,t) = \rho_o K_z^o \frac{\partial u_{\text{oce}}}{\partial z}(0,t) = F_{\text{oa}}(u_{\text{atm}}(0^+,t) - u_{\text{oce}}(0^-,t))$$

Boundary layer parameterization



typical vertical viscosity profile

$$F_{\mathrm{oa}}(\Delta U) = C_D(\mathbf{u}_\star)\,|\Delta U|\,\Delta U$$

with $\mathbf{u}_\star$ solution of

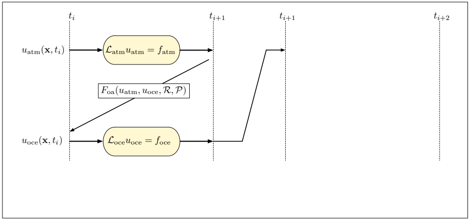
$$\frac{\Delta U}{\mathbf{u}_\star} = \frac{1}{k}\left[\ln\left(\frac{z_{\mathrm{atm}}}{z_0}\right) - \psi_m\left(\zeta(\mathbf{u}_\star)\right)\right]$$

Keywords: parameterization of Reynolds terms, K-profile schemes, Monin-Obukhov theory, bulk formulas...

Impact on the physics: more robust solution (2)

15-day simulation (60 6-hour time windows)

Usual method



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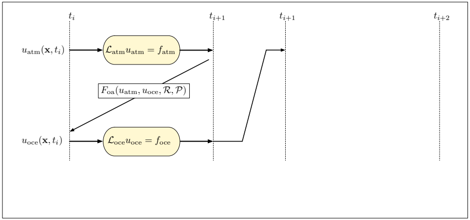
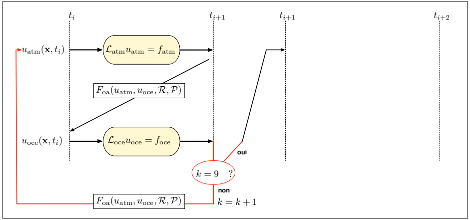
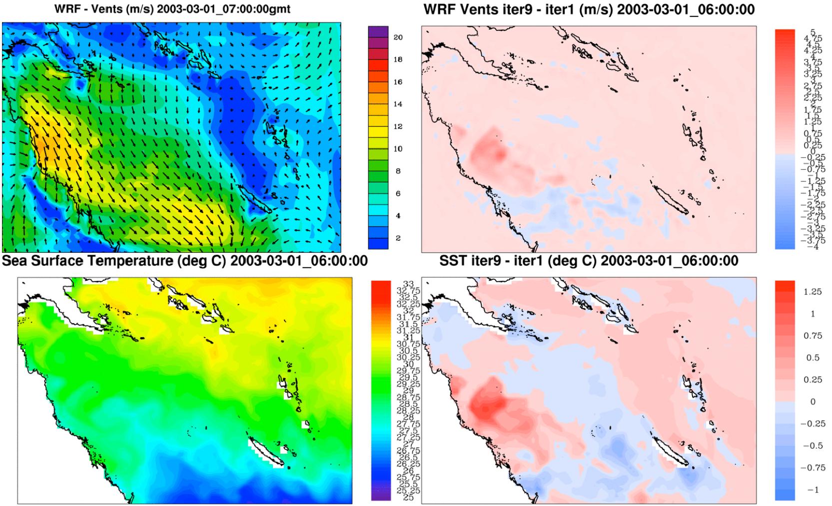


Schwarz method



Impact on the physics: more robust solution (3)

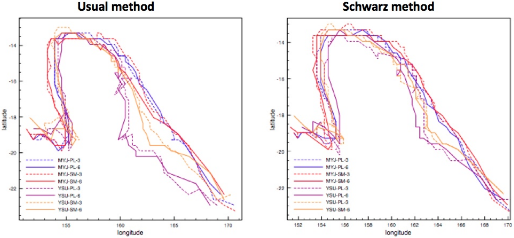


10-meter wind (m/s) and sea surface temperature (°C).

Impact on the physics: more robust solution (4)

To assess the robustness of the coupled solution: ensemble simulations w.r.t. uncertain system parameters

- PBL/SL: Mellor-Yamada-Janjic (MYJ) vs Yonsei University (YSU)
- Microphysics: Purdue Lin scheme vs Single-Moment 3-class scheme
- Length of the time windows: 6h vs 3h

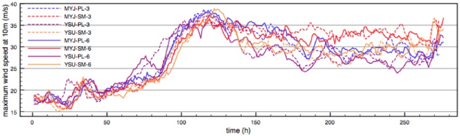


		3h	6h
1 itération	déviation moy	85km	94km
	déviation max	214km	238km
		3h	6h
9 itérations	déviation moy	66km	71km
	déviation max	167km	190km

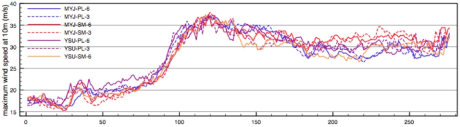
Trajectory of the cyclone

Impact on the physics: more robust solution (5)

Usual method



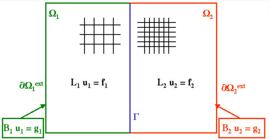
Schwarz method



		3h	6h
1 itération	déviation moy	2.49 m/s	2.46 m/s
	déviation max	3.03 m/s	3.19 m/s
		3h	6h
9 itérations	déviation moy	1.27 m/s	1.36 m/s
	déviation max	2.43 m/s	1.67 m/s

Intensity of the cyclone

Decreasing the cost: absorbing boundary conditions

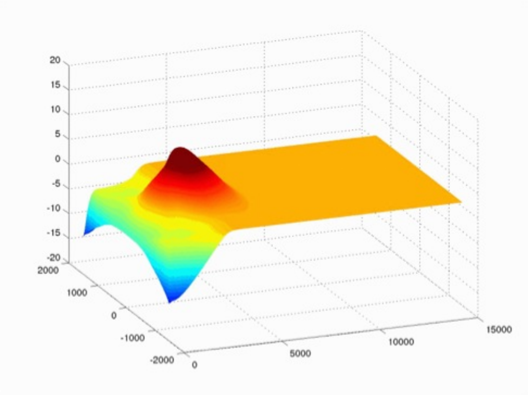


$$\begin{cases} L_1 u_1^{n+1} & = f_1 & \Omega_1 \times [0,T] \\ u_1^{n+1} & \text{given} & \text{at } t = 0 \\ B_1 u_1^{n+1} & = g_1 & \partial\Omega_1^{\text{ext}} \times [0,T] \\ C_1 u_1^{n+1} & = C_1 u_2^n & \Gamma \times [0,T] \end{cases} \qquad \begin{cases} L_2 u_2^{n+1} & = f_2 & \Omega_2 \times [0,T] \\ u_2^{n+1} & \text{given} & \text{at } t = 0 \\ B_2 u_2^{n+1} & = g_2 & \partial\Omega_2^{\text{ext}} \times [0,T] \\ C_2 u_2^{n+1} & = C_2 u_1^n & \Gamma \times [0,T] \end{cases}$$

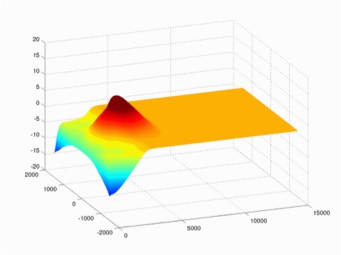
Systems satisfied by the errors:

$$\begin{cases} L_1 e_1^{n+1} & = 0 & \Omega_1 \times [0,T] \\ e_1^{n+1} & = 0 & \text{at } t = 0 \\ B_1 e_1^{n+1} & = 0 & \partial\Omega_1^{\text{ext}} \times [0,T] \\ C_1 e_1^{n+1} & = C_1 e_2^n & \Gamma \times [0,T] \end{cases} \qquad \begin{cases} L_2 e_2^{n+1} & = 0 & \Omega_2 \times [0,T] \\ e_2^{n+1} & = 0 & \text{at } t = 0 \\ B_2 e_2^{n+1} & = 0 & \partial\Omega_2^{\text{ext}} \times [0,T] \\ C_2 e_2^{n+1} & = C_2 e_1^n & \Gamma \times [0,T] \end{cases}$$

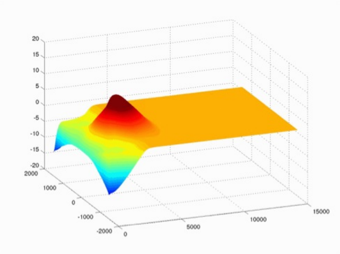
If one finds C_1, C_2 such that $C_1 e_2 = 0$ and/or $C_2 e_1 = 0$, then convergence in 2 iterations. $\longrightarrow$ **absorbing conditions**



shallow water - channel configuration (Martin, 2005)



Dirichlet-Dirichlet



optimized conditions

Solutions after 2 iterations

Some recent or ongoing works towards efficient interface conditions for ocean and atmosphere models

- Shallow water without advection (V. Martin, 2005)
- Shallow water with advection (V. Martin, E.B., on going work)
- Linearized primitive equations (E. Audusse, P. Dreyfuss and B. Merlet, 2009)
- Navier-Stokes (D. Cherel, A. Rousseau, E.B., on going work)
- Coupling between 3D Navier-Stokes and 2D shallow water (M. Tayachi, starting work with N. Goutal, V. Martin, A. Rousseau)
- 1-D advection-diffusion with variable and discontinuous coefficients $\rightarrow$ ocean-atmosphere coupling (F. Lemarié, L. Debreu and E.B., 2010; C. Japhet, on going work)
- ...