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Problems of Modeling Climate and Climate Change

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Abstract—The properties of the climate system as a physical object are considered. Major concepts of the mathematical theory of climate are stated, and the problems of constructing mathematical climate models are discussed. The results of reproducing the present-day climate are analyzed, and the sensitivity of the climate system to changes in the content of greenhouse gases is considered. Major directions are formulated in which the development of the mathematical theory of climate and of modeling climate and climate change is possible.

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1. INTRODUCTION

The central problem of the modern theory of climate is the prediction of its changes caused by anthropogenic activities. In view of specific peculiarities of the climate system, which are discussed below, this problem cannot be solved with the use of the conventional methods repeatedly tested in natural sciences. It can be stated that, at present, the principal methodological basis for solving this problem is numerical simulation of the climate system with the aid of climate models based on global atmosphere–ocean general circulation models. It is clear that the formulation of the climate models requires a comparison with real data and special-purpose field experiments in addition to observations carried out on a continuous basis. Analysis of the results of these experiments must enable the construction of increasingly more accurate models of specific physical processes determining the dynamics of the climate system. However, this approach is insufficient for solving the principal problem, namely, the problem of determining the sensitivity of the actual climate system to small external forcings.

In this study, an effort is made to consider major problems arising during solution of this principal problem. While dividing the paper into sections, we suggested that these sections could be read generally independently. For example, the sections on climate models and experiments with these models can be examined independently of the sections in which the mathematical theory of climate is described. It should be emphasized that most of the scientific community concerned with climate models are generally not interested in mathematical problems and treat a model as a specific finite-dimensional construction with a specific description of physical processes, thus reducing the study of the model's quality to a numerical experiment alone. However, as will be shown below, fundamental problems of the predictability of climate changes are completely or partially disregarded in this case.

1. CLIMATE SYSTEM AND CLIMATE

In the beginning of this section, it is appropriate to define two concepts:

(i) The climate system is the system combining the atmosphere, ocean, cryosphere, land, and biota.

(ii) Climate is an ensemble of states passed by the climate system during a sufficiently long time interval.

Several questions arise in the context of these definitions. First, what is the state of the climate system? A strict formulation of this term will be given below when the concept of an ideal model of the system will be defined. Here, we only note that the climate system is characterized by a finite set of parameters whose values at a fixed time determine its state. The set of parameters is finite because the number of molecules forming the system is finite. However, when describing the evolution of the climate system, researchers use differential equations with partial derivatives, whose solutions belong to an infinite-dimensional space, so that, in a general sense, it is necessary to prove the existence of a finite number of parameters controlling the state of the system.

In additional, the problem consists in the fact that, if the parameters are considered to mean, for example, temperature, pressure, velocity components, etc., on a certain sufficiently dense set of points, there has been no measured state of the climate system up to date. However, from some a priori considerations, it is possible to specify a certain number of parameters characterizing the climate system and to treat these parameters precisely as the desired set. For example, one can define that it is sufficient to characterize the climate system by one parameter—the global average surface temperature. Clearly, there are researchers that are interested only in this parameter and their ranks are sufficiently large that one-parameter models of the climate system should exist. However, in this case, a more important question arises on the ability of such models to predict anything, because it should be remembered that a necessary condition for the quality of any theory is its prediction possibilities.

Next, what does an ensemble represent? If the ensemble is considered to mean a set of states and a certain probability measure given on this set, it is necessary to have some quantitative characteristic determining the probability that the climate system may be located on a certain subset of the given set. In this case, no information is required on the probability of transition from one to another subset.

Finally, a more complicated question is associated with a quantitative definition of the concept of a sufficiently long time interval. Mathematically, it is convenient to take this interval as very long (infinite in the limit); however, in this case, no proper (internal) variability of climate can occur, and only climate changes under the effect of external forcings on the climate system can be considered. It is also possible to advance arguments in favor of other time intervals; however, it should be emphasized that, in any case, the concept of climate must be defined. Very frequently, climate is considered to mean some characteristics of a certain portion of the trajectory of the climate system—for a one-parameter climate model this characteristic will be, for example, the surface air temperature averaged over a definite time interval (say, 30 years).

The main problems of climate theory can be formulated as follows:

(i) reproduction of present-day climate (understanding of physical mechanisms of its formation),

(ii) assessment of possible climate changes under the impact of small external forcings (the problem of sensitivity of the climate system), and

(iii) prediction of climate changes.

It is assumed that the components of the climate system are continuous media, which are to be described with a definite accuracy. Specific postulates used to describe these media will be discussed below. The problems are discussed in order of increasing complexity because any prediction of the state of the system is directly related to the assessment of the degree of its stability in a certain sense. It is precisely this problem that is the central problem of mathematical theory of climate.

2. PECULIARITIES OF THE CLIMATE SYSTEM AS A PHYSICAL OBJECT

The climate system as a physical object possesses a number of specific properties.

(i) The principal components of the system—the atmosphere and the ocean—can be treated geometrically as thin films because the ratio of the horizontal to the vertical scale amounts to about 0.01–0.001. Thus, the system is quasi-two-dimensional; however, the vertical stratification in density is of great importance and large-scale vertical motions are responsible for baroclinic transformations of energy. The characteristic time scales of energy-significant physical processes lie in a range between 1 h and tens and hundreds of years. As a consequence, laboratory modeling of such a system presents a severe problem.

(ii) No purposeful physical experiment can be carried out with the climate system. Indeed, it is impossible to pump up the system with, for example, carbon dioxide and measure the resulting effect, all other conditions being the same.

(iii) Researchers have only short series of observational data on individual components of the system under study at their disposal.

Clearly, there are many other important properties of the climate system that should be considered; however, even the aforementioned properties allow the conclusion that the principal means for studying the climate system (more exactly, the problem of predictability and prediction of climate) is mathematical (numerical) modeling. Experience in recent years shows that the main results of climate theory have been obtained on the basis of constructing and using global climate models.

3. MATHEMATICAL THEORY OF CLIMATE

The first question to be answered relates to the goals and methods of the mathematical theory of climate. The methods of the mathematical theory of climate are methods of the theory of dynamical systems. To apply the methods of this theory to studies of the actual climate system, it should be assigned a certain mathematical object that represents an idealization of the system of interest and can be referred to as its "ideal" model. It is suggested that such an ideal model exists and the observed dynamics of the climate system is a realization of the trajectory generated by this model. It is also assumed that this model belongs to the class of dynamical dissipative systems that is described formally by the following system of equations:

$$\frac{\partial \varphi}{\partial t} + K(\varphi)\varphi = -S\varphi + f, \qquad \varphi|_{t=0} = \varphi_0, \quad \varphi \in \Phi. \tag{1}$$

In system (1), φ is the vector function of the parameters of the climate system (u, v, w, ...), which depends on spatial coordinates and time; $K(\varphi)$ is the "dynamical" operator of the problem; S is the operator that describes the dissipation of the system; and f is an external forcing. We will assume that the system is reduced to the form in which its energy can be expressed as the quadratic form $E \equiv (\varphi, \varphi)$; i.e., we will also assume that $\varphi \in \Phi$, where Φ is the Hilbert space with the scalar product $(\cdot,\cdot)$. By definition, the space Φ is the phase space of system (1). The system under study is considered to be open, and its effect on the external energy flux is negligibly small. We will assume that the solution to the system φ is deterministic; i.e., φ exists and is unique at a given φ_0 on an arbitrarily long time interval T.

Here, a remark should be made. The formulation of the problem given in (1) with the external forcing f is valid to a certain accuracy only on a finite interval T because the Sun loses energy. Consequently, the solvability of (1) on an infinite time interval is generally not required, altthough it is useful (see below).

Further, as was already noted, $\varphi \in \Phi$, where Φ is the infinite-dimensional Hilbert space. We will assume that it is separable; i.e., a countable basis $\{\varphi_i\}$ can be introduced in this space, so that the function φ can be expanded in this basis:

$$\varphi = \sum_{i=1}^{\infty} \alpha_i \varphi_i.$$

The Fourier coefficients α_i can be regarded as the coordinates of φ in the space Φ similarly to the conventional geometric coordinates. If φ_i is a function of spatial coordinates alone, α_i is a function of time. Then, the function φ at any time t_0 can be treated as a point in the space Φ with the coordinates $\{\alpha_i(t_0)\}$ and the solution $\varphi(t)$ will represent a curve in this space with varying t. This curve will be referred to as a trajectory.

The meaning of the qualitative theory of differential equations lies in an attempt to predict the qualitative behavior of the trajectory generated by this system without knowledge of the trajectory itself on the basis of the form of system (1). More strictly, the theory is bound to predict the behavior of the trajectory on sufficiently long (infinite in the limit) time intervals. The corresponding results are usually formulated in terms of a global attractor. We will assume that system (1) possesses a global attractor, which represents a certain set in the phase space such that all trajectories emerging from any point of the space Φ are attracted to this set as time passes. We will also assume that the global attractor is a compact set. As a rule, this is the case even if Φ is infinite-dimensional. Finally, the global attractor is minimum in a certain sense; i.e., there is no another set within the global attractor that would possess all the properties of the global attractor.

Mathematically rigorously, these conditions are formulated in the following form [1, 2]. The set $A \subset \Phi$ is called the global attractor of the subgroup $S(t)$, $t \geq 0$, if

(i) A is compact;

(ii) A is invariant, i.e., $S(t)(A) = A$, $\forall t \geq 0$; and

(iii) A attracts each bounded set $B \subset \Phi$.

The above definition does not contradict the fact that the global attractor can contain local attractors, which attract trajectories not from the entire space Φ but from its subset.

Thus, the entire dynamics of system (1) can conventionally be divided into two stages: motion toward the attractor and motion on the attractor and in its vicinity. During motion toward the attractor, contraction of the phase volume generally occurs: a volume of an arbitrary dimension contracts to the volume of the attractor, which has a finite dimension. (The attractor's volume is equal to 0 in the sense of the volume in the space Φ.) During motions on the attractor, when trajectories emerge from the vicinity of its arbitrary point, the volume cannot contract systematically, because a global attractor involved in the initial attractor would exist otherwise. This is a key point in evaluating the dimension of attractors.

Since the attractor is finite-dimensional in a certain sense, it seems logical to write down a finite-dimensional system of equations that would be dynamically equivalent on the attractor to the initial infinite-dimensional system given in (1). However, this process is virtually impossible because of a very complicated, often fractal, structure of the attractor. This can be done if the system possesses the so-called inertial manifold—a smooth finite-dimensional attracting set containing the attractor. In this case, from the proof of the existence of the inertial manifold, it follows that, in a general sense, the problem of closure of some scales of motion through other scales can be solved. However, it should be remembered here that of fundamental importance are the answers to the questions as to what scales can be parameterized and to what extent these scales are comparable to molecular scales, because the transition from finite-dimensional to infinite-dimensional systems occurs precisely at the molecular level. The reverse transition has meaning only if the resulting scale is much greater than the molecular scale.

The next assumption lies in the fact that the dynamics of the climate system occurs on its attractor; i.e., the climate system has enough time to be attracted to its attractor. For qualitative analysis of the dynamics, it is most simple to turn to state-of-the-art models of the climate system, which adequately describe the present-day climate. For simplicity, we will consider atmospheric models alone.

Experience in short- and long-range weather forecasting indicates that the trajectory of the atmosphere is unstable in the Lyapunov sense: whatever the small error contained in the initial data, there is always the time T at which the error reaches a significant value. If we assume that the trajectory lies on the system's attractor, the "energy" of the error will be limited by the "size" of the attractor. The rate of divergence of trajectories was also evaluated from observational data. It should be noted that, from the practical standpoint, this problem is very complicated because it is very difficult to find two closely located points in the phase space owing to the fact that the time of trajectory return into the vicinity of the initial point is very large. The existence of this time follows from the Poincaré theorem [1, 2].

Unstable trajectories enclosed in a bounded volume (attractor) generate dynamical chaos. Dynamical chaos implies that if a bundle of trajectories is allowed to escape from a small vicinity of a point φ_0, these trajectories will run away; however, the closed volume will not allow them to go away at infinity and they will be mixed in a complicated way. The characteristic time of running away will be determined by positive Lyapunov exponents: their number yields the number of directions along which the trajectory is unstable. Since the phase volume on the attractor does not regularly contract but there are directions of expanding this volume, there must be contracting directions along which the volume must on average contract to the extent that it extends along unstable directions. The term "on average" here means that the Lyapunov exponents are asymptotic characteristics. Following the above considerations, one can infer that the number of positive Lyapunov exponents characterizes the attractor's dimension: if this number is large, the attractor's dimension is also large.

Let us turn to the goals of the mathematical theory of climate. What problem is to be solved on the basis of this theory? Ideally, it is necessary to construct the theory of the climate system's sensitivity to small external forcings that would yield a constructive method to calculate climate changes under the influence of these forcings. Since the basic method of study of the climate system is mathematical (numerical) simulation, it is seemingly clear that mathematical models should be used to solve this problem. However, a question arises: What and to what accuracy must the climate model reproduce in order that its sensitivity to various small external forcings would be close to the climate system's sensitivity? To answer this question, it is necessary to find the operator of the model's response to small external forcings in an explicit form. This operator can be constructed in principle if the model's attractor (as a set of states) and the measure on it depend continuously on the external forcing. However, this statement virtually cannot be proved in a general case; therefore, we will use the procedure proposed in [3], which can be called the ε-regularization technique.

The essence of this technique lies in that a small δ-correlated (in time) Gaussian random process is added to the right-hand side of the model. The inclusion of this process can be considered quite substantiated because all physical mechanisms responsible for the formation of the sources of energy and its dissipation are never known exactly. Such a simple procedure results in that the probability measure becomes smooth and a differential equation for studying its evolution (the so-called Fokker–Planck equation) can be written.

Thus, we will assume that the climate system is governed by the following finite-dimensional system of equations:

$$\frac{du_i}{dt} = B_i(u) + \varepsilon_i(t),$$
$$u_i\big|_{t=t_0} = u_{i0}, \quad u \in R^N, \quad i = 1, \ldots, N, \tag{2}$$
$$\langle \varepsilon_i(t)\varepsilon_j(t')\rangle = 2d_{ij}\delta(t-t'),$$

where $\langle\cdot\rangle$ is the sign of averaging over an ensemble ε. For simplicity, Eqs. (2) can be treated as a finite-dimensional approximation of system (1). The methods of constructing such approximations will be discussed below. We will also assume that $d_{ij} = d\delta_{ij}$, where δ_{ij} is the Kronecker delta. Then, the following Fokker–Planck equation can be written for the probability density ρ of the process u:

$$\frac{d\rho}{dt} + \mathrm{div}B(u)\rho = d\Delta\rho. \tag{3}$$

The function ρ must satisfy the nonnegativeness condition $\rho \geq 0$ and the normalization condition $\int \rho\, du = 1$. In order to construct the response operator correctly, it is necessary to prove (i) the existence of a steady-state solution to Eq. (3), (ii) its uniqueness in the class of probability densities, and (iii) its asymptotic stability. These properties of the steady-state solution to Eq. (3) are proved if $u \in U$, where U is a compact manifold without a boundary [3]. In a general case of the phase space R^N, the problem remains unsolved.

Consider a disturbed equation

$$\frac{du_i^{(1)}}{dt} = B_i(u^{(1)}) + \varepsilon_i(t) + \delta f,$$
$$u_i^{(1)}\big|_{t=t_0} = u_{i0}. \tag{4}$$

Let $\delta u = u^{(1)} - u$. Now, it is necessary to find the relation between $\langle\delta u\rangle$ and δf. This relation can be established if δf is small [4]. Correct to second-order terms, it can be written that

$$\langle\delta u_i\rangle = \sum_j \int_0^t \langle u_i(t)\frac{1}{\rho}\frac{\partial\rho}{\partial u_j}(t')\rangle dt'\delta f_j. \tag{5}$$

This relation is converted to its simple form if the dynamics of the system represents a stationary random process and ρ obeys the Gaussian distribution:

$$\langle \delta u(t) \rangle = \int_0^t C(\tau) C^{-1}(0) d\tau \delta f, \qquad (6)$$

where $C(\tau)$ is the covariance function with shift τ:

$$C(\tau) \equiv \langle u(t+\tau) u^T(t) \rangle.$$

Relation (6) is the so-called fluctuation–dissipation relation that was established earlier for regular systems [4]. From (6), it follows that, under the assumptions made above, one can, in principle, calculate the operator of the response of a real nonlinear system to small external forcings if the observed trajectory of the system has a sufficient length.

The validity of relation (6) has been verified for different atmospheric models. With a high accuracy, it is satisfied for a barotropic and a two-layer baroclinic global atmospheric model if the external forcing is specified in the subspace stretched over the principal natural orthogonal vectors—the eigenvectors of the operator $C(0)$ [5, 6]. With a good accuracy, this relation also holds for global atmospheric general circulation models [7, 8]. Specifically, in [8], the NCAR (National Center for Atmospheric Research) atmospheric general circulation model [9] was used. The response operator was calculated from a trajectory two-million-days long. Direct numerical experiments on deriving the response were performed for a time of about one hundred thousand days. The total results of analysis of about 300 experiments were published in [8]. As an illustration, Fig. 1 presents the results of four numerical experiments on reproducing the linear part of the model's response (in the temperature field at a height of ≈925 hPa) to a vertically extended equatorial thermal source with a heating maximum located at 60° E, 150° W, 105° W, and 15° W, respectively (see the left-hand column from top to bottom). The right-hand column shows the responses obtained from the constructed approximate response. The fields depicted in Fig. 1 indicate that the results of calculations with the response operator and of direct modeling are very close to each other.

To conclude this section, we emphasize that an important result of the validity of fluctuation–dissipation relations on the attractors of climate models is the establishment of the fact that the linear operator of the response is determined not only by climatic characteristics of the attractor (see the definition of climate)—the operator $C(0)$, – but also by dynamics on the attractor—the operator $C(\tau)$. This finding implies that, for a correct reproduction of the response, climate models should also be identified in terms of the closeness of the generated dynamics to the actually observed dynamics of the climate system.

4. MATHEMATICAL MODELS OF THE CLIMATE SYSTEM

Let us discuss the main propositions on which the construction of state-of-the-art climate models is based. The state-of-the-art climate models are based on a combined atmosphere–ocean general circulation model. A central direction of their development is associated with an increasingly accurate description of all physical processes participating in climate formation. This direction appears to be reasonable because, as is shown in the previous section, in order to correctly describe the climate system's response (even its first moment) to small external forcings, it is necessary to adequately reproduce not only climate itself but also the dynamics on the attractor of the climate system (the probability of transition of the climate system from one state to another).

A number of principles underlie the construction of state-of-the-art climate models. It is suggested that the equations of classical equilibrium thermodynamics are locally valid. It is also assumed that the Navier–Stokes equations for a compressible fluid can be used to describe the dynamics of the atmosphere and ocean. Since the Reynolds equations, which represent the Navier–Stokes equations averaged over some spatial and temporal scales, are used in such models in view of primarily computational potentialities, it is believed that a fundamental feasibility of their closure exists. The closure procedure suggests that the effects of processes on subgrid scales (scales smaller than the scale of averaging) can be expressed via the characteristics of processes on larger scales. These processes include (i) the transfer of shortwave and longwave radiation, (ii) phase transitions of water and the process of local precipitation formation, (iii) convection, (iv) boundary and inner turbulent layers (with some of their characteristics described explicitly), (v) small-scale orography, (vi) wave resistance (interaction of small-scale gravity waves with the main stream), (vii) small-scale dissipation and diffusion, and (viii) small-scale processes in the active layer of land. Finally, for the description of large-scale atmosphere–ocean motions, the hydrostatic approximation is valid. This approximation implies that the vertical pressure gradient is balanced by the gravity force. The use of the hydrostatic approximation requires additional simplifications, such as a constant radius of the Earth and the disregard of the components of the Coriolis force with a vertical velocity component, so that the law of energy conservation is satisfied in the system of equations in the absence of external sources of energy and dissipation. The equations of hydrothermodynamics of the atmosphere and ocean, closures of subgrid-scale processes, and boundary conditions are discussed thoroughly in [7].

In accordance with the ideas presented in the previous section, it is necessary to prove some statements

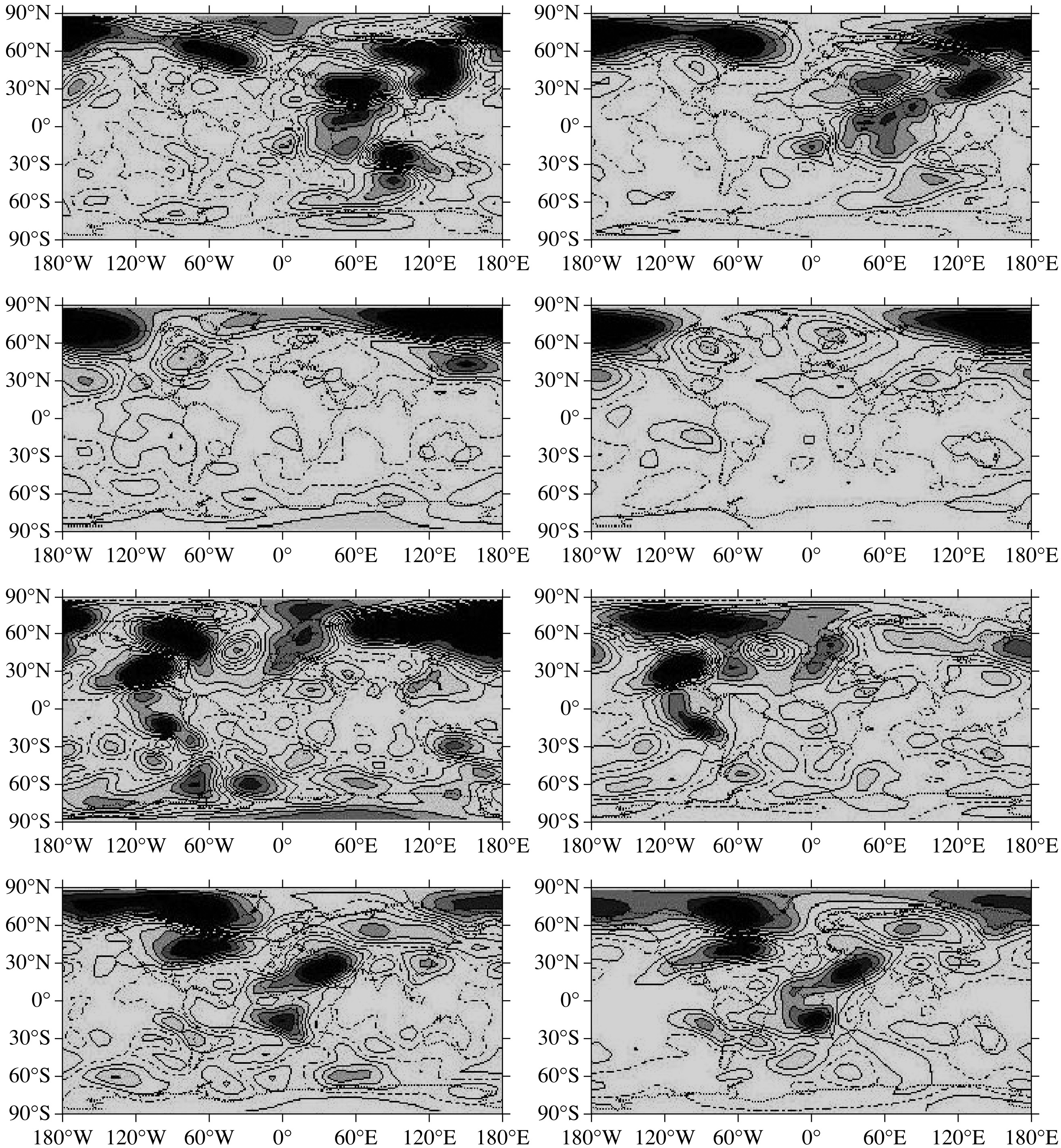

Fig. 1. (Left) Linear part of the model's response to a vertically extended anomaly of temperature at the equator and (right) response obtained via fluctuation–dissipation relations. The responses are shown in the field of temperature (K) at a height of 925 hPa for heating with maxima at the points (from top to bottom) 60° E, 150° W, 105° W, and 15° W.

for the system of partial differential equations describing the climate system's model.

I. The global theorem of solvability on an arbitrarily large time interval t.

Unfortunately, there is presently no such a theorem in a spherical coordinate system with "correct" boundary conditions. This is not a consequence of the absence of such theorems for three-dimensional

Navier–Stokes equations. The equations of the state-of-the-art climate models have a dimension of 2.5 because the hydrostatic equation is used instead of the full third equation of motion.

II. Existence of a global attractor.

This statement is presently proved for the systems of equations of form (1) from the previous section provided that S is strictly positive definite operator:

$$(S\varphi, \varphi) \geq \mu(\varphi, \varphi), \quad \mu > 0.$$

The problem lies in that this cannot be stated in a general case because the continuity equation for a compressible fluid is nondissipative.

III. The attractor's dimension.

Constructive estimates made for the dimension of the attractors for the models of this class are very rough. These estimates represent upper bounds, which are generally unsuitable for the theory discussed in the previous section.

5. FINITE DIMENSIONAL APPROXIMATIONS

Clearly, it is virtually impossible to obtain analytic solutions to complicated nonlinear equations of hydrothermodynamics of the atmosphere and ocean for arbitrary initial data; therefore, their approximate solutions are being sought with the aid of various finite dimensional approximations. Let there be a quadratic law of energy conservation (or a law that can be made quadratic via some nonlinear transformations of the desired functions) in system (1) of Section 3 in the absence of dissipation and of external and internal sources of energy. From the analysis made Section 3, it follows that, on average, the cancellation of dissipation and energy sources must occur on the attractor of system (1). This result implies that finite dimensional approximations should be constructed so that a quadratic conservation law—an analogue of the initial law—is satisfied in the absence of dissipation and energy sources. In this case, this conservation law leads automatically to a computational stability of the solution of the difference problem if the stability is considered to mean a continuous dependence of the solution's norm on both the norm of the right-hand side and the norm of the initial data.

At the same time, this requirement is insufficient for construction of difference schemes for climate models. It is significant that, unlike the problems of weather forecasting, where it is necessary to reproduce the solution of the problem on a finite time interval, the problems of climate require that the attractor of the initial model be approximated as a set and a measure on it or a statistical steady-state solution (see Section 3). The proof of a global solvability for finite dimensional climate models and the proof of the existence of a global attractor for them present no special difficulties [1, 2]. However, the problem lies in the proof of the convergence of the attractors of finite dimensional approximations to the attractor of the initial model as the approximation's parameters approach zero. The complexity of the problem lies also in the selection of the metric in which the convergence is considered. Constructive estimates for the aforementioned convergence in "useful" (Hausdorff) metrics are presently absent and represent an important and interesting problem for numerical mathematics. Since there are no convergence theorems, an approach based on the approximation of most significant physical processes participating in climate formation is used in modeling the climate system. Some examples of such processes are presented below.

Since the atmosphere and the ocean are quasi-two-dimensional, the transfer of energy across the spectrum in these media is governed by the laws of a two-dimensional fluid. It is well known that there are two quadratic invariants in an ideal incompressible two-dimensional fluid: energy and enstrophy (vorticity squared). Moreover, the distribution of energy in the inertial range is determined by the transfer of enstrophy toward high wave numbers. Recall that the inertial range is the range of scales where energy dissipation and generation are virtually absent and the main process is the transfer of energy across the spectrum. In order to satisfy this condition in a numerical model, it is necessary to construct finite dimensional analogues so that, in two-dimensional asymptotics, there will also exist finite dimensional analogues of energy and enstrophy that are invariants in the absence of dissipation and energy sources.

However, note that the measurements performed in the past decades [10, 11] have shown that the atmosphere has fundamental features that differreniate its evolution from the behavior of a quasi-two-dimensional fluid. The generation of energy in the atmosphere occurs on synoptic scales as a consequence of the occurrence of baroclinic instability. On scales greater than the synoptic scales, the inertial range is absent and the distribution of energy over the spectrum on these scales is determined by the ratio between the characteristic time of energy dissipation in the boundary layer and the characteristic time of nonlinear interactions. On scales smaller than the synoptic scales, the inertial range exists and, according to the theory of two-dimensional turbulence, the distribution of energy in this range has the form k^{-3}, where k is the spatial wave number. However, starting with a scale of about 800 km, the distribution of energy obeys the law $k^{-5/3}$, as in the Kolmogorov three-dimensional turbulence, although the atmosphere is evidently quasi-two-dimensional on these scales. This paradox is presently explained either by superdissipation on fronts [12] or by breaking of gravity waves in the stratosphere [13].

Further, the law of conservation of angular momentum about the Earth's rotation axis actually determines the distribution of wind velocity near the Earth's surface (the presence of trade winds). The law of conservation of entropy in the adiabatic approximation is also of importance. Additionally, specific physical phenomena such as cyclogenesis, 30–60-day oscillations in the tropics, the propagation of quasi-stationary waves, and many other processes responsible for climatic characteristics are noteworthy. A correct reproduction of cyclogenesis requires a close spectral approximation of some linear operators (in eigenvalues and singular values). The solution of the transport equations for trace gases is of special interest. These gases are characterized by large spatial gradients, which impose a very strong requirement on the condition of monotonicity of difference schemes.

To conclude this section, we touch on one more current problem in numerical mathematics—the problem of mapping computational algorithms onto computer architecture. It is well known that the development of computer engineering and computational algorithms is presently associated with parallel computations. Modern estimates of computational algorithms may differ substantially from the established estimates associated with estimates of sequential computations. Frequently, a researcher who works on massively parallel computing systems has to select an algorithm that is not the most elegant and the most efficient for sequential computations but that is easily subject to multisequencing. Since a tremendous number of arithmetic operations are used in the course of solving climatic problems, which are of great importance, it seems reasonable to design computing systems oriented immediately toward the solution of these problems.

6. REPRODUCTION OF THE PRESENT-DAY CLIMATE

Constructed in Section 3, the linear operator of response to a small external forcing δf,

$$\langle \delta u \rangle = \int_0^\infty C(\tau) C^{-1}(0) d\tau \delta f,$$

does not depend explicitly on the mean of the vector u on the model's attractor. An implicit dependence exists and follows, for example, from the linear theory of low-frequency variability. In the context of this theory, it is possible to construct the linear equation

$\frac{d\varphi'}{dt} + A\varphi' = f'$, where the operator A can be treated as the problem's operator linearized about the mean state with the use of the corresponding closure procedure for nonlinear terms and where f is a Gaussian process δ-correlated in time. The covariance matrix of this equation will have the form

$$C(\tau) = e^{A\tau} C(0), \quad \tau > 0,$$

so that

$$C(\tau) C^{-1}(0) = e^{-A\tau}$$

and, consequently, low-frequency fluctuations in the field φ' are completely determined by the mean state (if the closure procedure takes place). Thus, even in the context of the problem of the climate system's response to small perturbations, it is necessary to reproduce not only the internal variability of climate but also its mean state.

This problem has been considered within the framework of the Atmospheric Model Intercomparison Project (AMIP), which has revealed many key mechanisms responsible for climate formation (http://www-pcmdi.llnl.gov/amip). At the same time, the AMIP can also be viewed as a program of study of the sensitivity of an "ideal" atmospheric model to the level of description of different physical processes. In particular, during modeling of global climate, it is necessary to reproduce a wide spectrum of its characteristics: seasonal and monthly means, intraseasonal variability (monsoon cycle, storm-track parameters, etc.), climatic variability (its predominant modes, phenomena like El Niño and Arctic Oscillation), and others. Among the problems of modeling regional climate are the reproduction of its characteristics with a high degree of detail, study of hydrologic-cycle features, estimation of the possibility of extreme phenomena, and studies of the consequences of regional climatic changes for the environment and socioeconomic relations. An important output of this program has been the solution of the following problems: (I) description of the present-day climate (1979–1999), (II) study of the nature of monsoon circulation, (III) study of the response of atmospheric circulation to an El Niño event, (IV) study of the role of soil processes in the formation of atmospheric dynamics, and (V) study of the interaction of radiation with cloudiness related to superabsorption in clouds. Among other interesting problems, one can note the modeling of (i) the stratosphere and mesosphere, (ii) the negative trend of temperature near the mesopause during the past three decades, and (iii) the role (in this process) of increasing carbon dioxide concentration and decreasing ozone concentration in the stratosphere. The reproduction of the El Niño statistics with a coupled model of the atmosphere–ocean general circulation seems to be a very important problem. Theoretically, this system can be treated as a tropical oscillator with a stochastic external forcing (or with an external forcing depending on time).

The recent intercomparison of atmospheric general circulation models made within the framework of

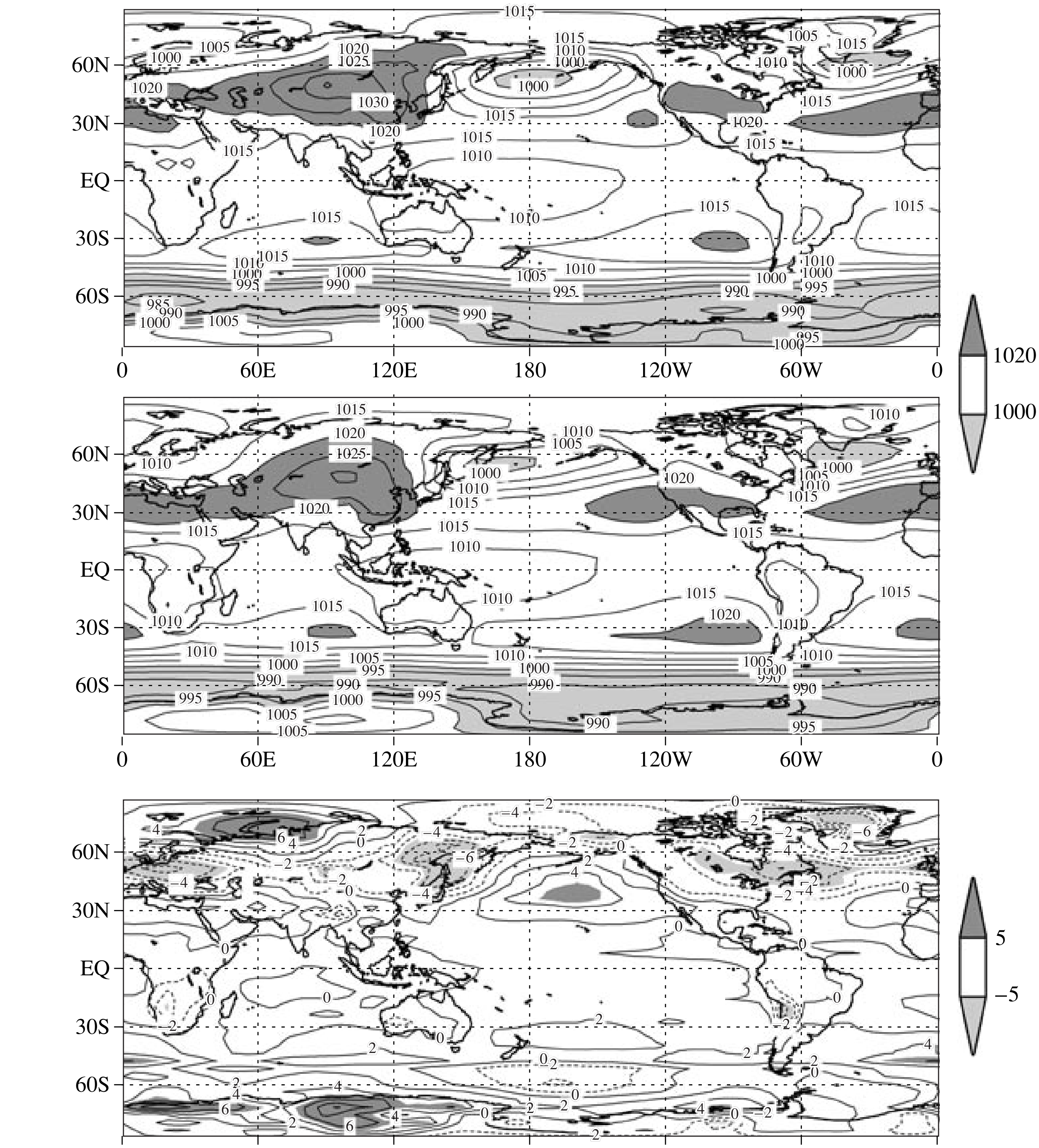

Fig. 2. Sea-level pressure (hPa) in winter: (from top to bottom) NCEP data, modeling results, and the difference between them.

AMIP II has shown that the best of these models are presently capable of reproducing the main features of the observed atmospheric circulation with good accuracy. Errors in reproducing many climatic quantities with such models are only slightly greater in value than the uncertainties with which these quantities are determined from observations. At the same time, there are also systematic errors in climate reproduction, which are inherent in virtually all of these models. The most complete analysis of climate reproduction with the models participating in AMIP II can be found at http://www-pcmdi.llnl.gov/amip.

The quality of state-of-the-art atmospheric general circulation models can be illustrated through the use of the results of reproducing some features of atmospheric circulation with the model of the Institute of

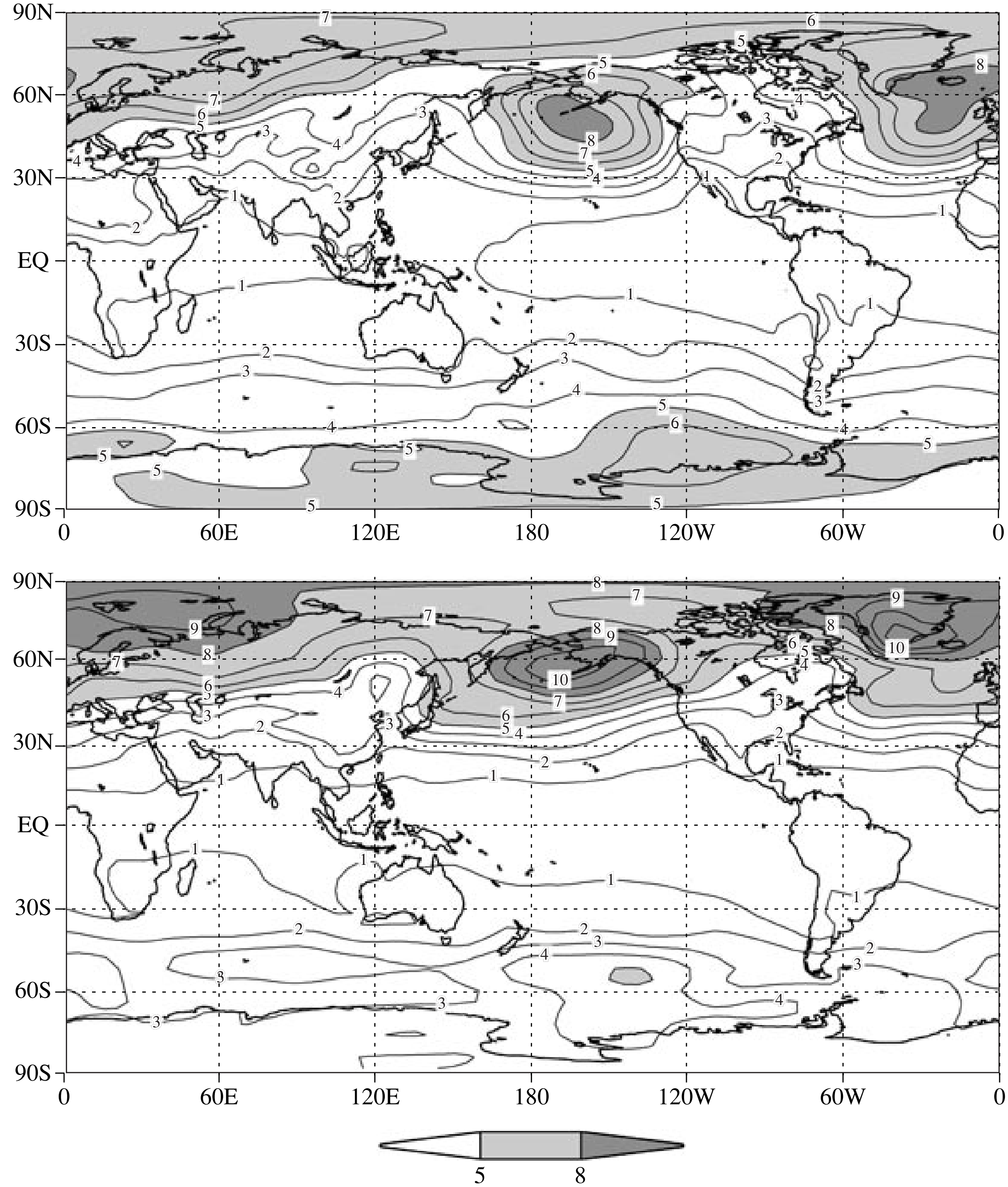

Fig. 3. Rms deviation of winter-mean pressure (hPa) from (top) the NCEP data and (bottom) the model.

Numerical Mathematics of the Russian Academy of Sciences (INM RAS) [7]. Figures 2–5 present some results of a numerical experiment performed with this model within the framework of the AMIP II scenario. The model, which is characterized by the resolution $5° \times 4°$ in longitude and latitude and 21 levels in the vertical, was integrated for 17 years. The time behaviors of the sea surface temperature and sea-ice boundaries observed during 1979–1995 were specified as the boundary conditions at the Earth's surface. The temperature of land was calculated in accordance with the equation of its thermal balance.

Figure 2 shows the geographic distributions of sea-level pressure in the winter season (December–February) that are constructed from the NCEP reanalysis data and modeling results and their difference. As is seen from the figure, all major "centers of action" are well reproduced by the model, including their locations and pressure values. Nevertheless, the error of

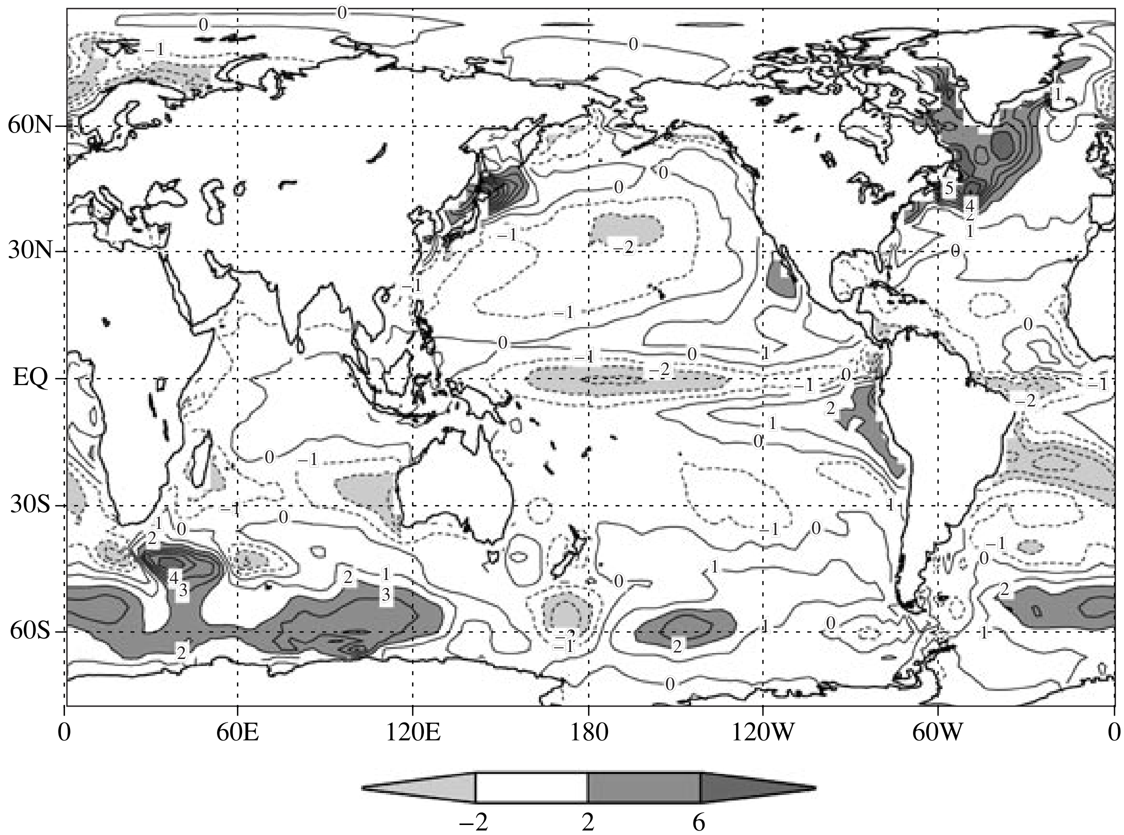

Fig. 4. Annual mean error of reproduction of sea surface temperature. The isoline interval is 1°C.

pressure reproduction reaches 4 to 6 hPa in some regions.

The low-frequency variability of atmospheric circulation in the model is shown in Fig. 3, which depicts the rms deviation of the monthly mean pressure from the climatic annual cycle for winter constructed from the NCEP data over 51 years and from modeling results over 17 years. The model adequately reproduces the maxima of variability in the winter hemisphere in the northern Pacific and Atlantic. The rms pressure deviation there, as well as over the Northern Hemisphere on the whole, is about 10% greater for the model than for the NCEP data. In the summer hemisphere, in contrast, the variability in the model is underestimated by 10 to 20% because of insufficient resolution of synoptic vortices.

Analysis of climate reproduction with an atmospheric model of higher horizontal resolution (2.5° × 2° in longitude and latitude) indicates that, in the summer hemisphere, the amplitudes of high-frequency and low-frequency variability become close to the observed amplitudes. However, systematic errors in reproducing the mean state of climate and winter-hemisphere variability do not generally decrease with increasing spatial resolution. This finding testifies either to the necessity of a more accurate adjustment of model parametrizations or to the necessity of introducing physical mechanisms not taken into account. Universal experience in modeling also basically confirms that systematic errors in reproducing the climatic mean state of the atmosphere depend only slightly on the spatial resolution of the model used.

Many key mechanisms responsible for climate formation have been revealed as a result of AMIP performance. This project has been developed in the Coupled Model Intercomparison Project (CMIP). During CMIP performance, the emphasis was placed on the reproduction of the surface temperature and sea-ice distribution because these characteristics were considered to be specified external parameters in the AMIP experiments. Below, the results of an 80-year numerical experiment on reproducing the present-day climate with the INM RAS coupled model of the atmosphere–ocean general circulation will be presented.

The annual mean error of reproducing the sea surface temperature is calculated as the difference between the results of the coupled model and the observational data from [14] and is presented in Fig. 4. As is seen in the figure, the temperature is somewhat underestimated (by 1 to 3°) in the equatorial Pacific as a consequence of an overestimated intensity of upwelling. This result is characteristic of

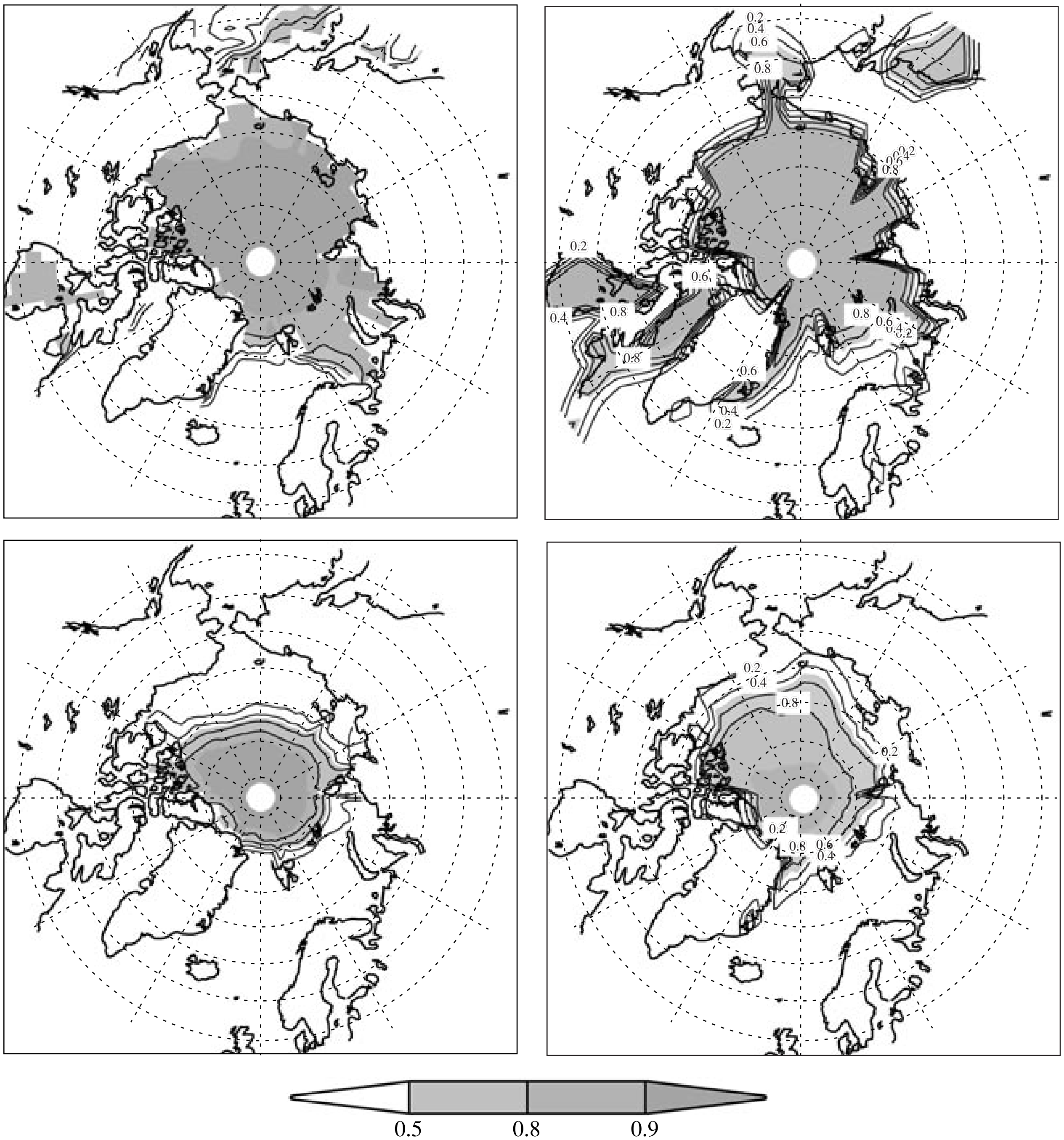

Fig. 5. Mean sea-ice concentration in the Northern Hemisphere in (top) March and (bottom) September from (left) modeling results and (right) observational data. The isoline interval is 0.2. The regions with an ice concentration above 0.5 are shaded.

almost all of the state-of-the-art models. Temperature overestimation by 2 to 6° occurs for the northwestern Atlantic and for the region near Japan. This result is associated with errors in reproducing the locations of the warm Kuroshio and North Atlantic currents. In the middle latitudes of the Southern Hemisphere, temperature overestimation is due most likely to an insufficiently accurate reproduction of the components of the radiation balance on the surface. On the whole, the integral surface temperature in the model (with consideration for land and sea ice) is 14.7°C, which is close to a value of 15°C estimated from observational data.

Figure 5 shows the distributions of sea-ice concentration in the Northern Hemisphere for March, when the amount of ice is maximum, and for September, when the amount of ice is minimum. For comparison, this figure also presents the corresponding data used in the AMIP II experiments and averaged over 1979–1995. For March, the ice area in the model is 10–15% smaller than the observed ice area because ice does not form near the eastern coast of Greenland and between Greenland and Canada. At the same time, according to the model's data, the Barents Sea freezes somewhat more strongly than according to observational data. These results appear to be associated with limitations of reproduction of the ocean circulation in high latitudes. For September, the amount of ice in the Arctic Ocean from the model is 20–30% smaller than that from observational data. Extremely intense thawing of ice occurs near Alaska and East Siberia, a phenomenon that is explained by surface-temperature overestimation for northern Siberia and Alaska as a result of an insufficiently accurate description of the heat balance on the surface.

7. SENSITIVITY OF THE CLIMATE SYSTEM TO CHANGES IN THE CONTENTS OF GREENHOUSE GASES

Diagnostic studies of the surface air temperature indicate the following: (i) for the past 30 years, marked changes have occurred in the surface air temperature averaged over decades—it has increased; (ii) maximum winter temperature changes are observed in Siberia and northwestern Canada; (iii) summer temperature changes are substantially smaller; and (iv) the sea surface temperature of the North Atlantic has not increased but even decreased. The question arises as to what the cause of these changes is. Do these changes result from proper oscillations of the climate system's parameters or do they result from anthropogenic impacts associated, for example, with increasing concentrations of carbon dioxide and sulfate constituents in the atmosphere? During analysis of the response of the climate system to disturbances of such a kind, it is expedient to use the notions of "dynamic" response and "radiation" response.

The authors of [15] studied the response to an increased carbon dioxide in the atmosphere on the basis of comparison of two computations performed for 80 years by the CMIP2 scenario (see http://www-pcmdi.llnl.gov/projects/cmip/index.php). In the first (control) experiment, the concentration of atmospheric CO_2 was taken to be invariant and equal to the concentration observed at the end of the 20th century. In the second experiment, the CO_2 concentration was increased by 1% per year. It has been shown that the principal role in the total response of the system to an increased concentration of atmospheric carbon dioxide is played by the radiation response. This phenomenon is manifested in the fact that the sensitivity of the climate system to increased CO_2 is determined primarily by the amount of heat expended in ocean heating and by the extent to which the Earth's radiation balance changes as a result of changes in the cloud amount during climate changes. At the same time, as is shown in Section 3, where the main propositions of the theory of sensitivity of the climate system to small external forcings are presented, it is also necessary to adequately reproduce the dynamic response, whose principal component is the Arctic Oscillation.

This section presents some results from study [16], devoted to numerical experiments with the INM RAS climate model on reproduction of climate changes in the 20th century and estimation of possible climate changes in the 21st and 22nd centuries in accordance with three scenarios of changes in the contents of greenhouse and other gases [17]. This version of the model was implemented on a 32-processor cluster Intel Itanium, and computations for ten model years on eight processors take 24 h. In Russia, such experiments have been performed for the first time. We will consider the following experiments out of those discussed in [16].

(1) Experiment on reproduction of the climate of the 20th century. In the course of this experiment, real temporal behaviors of changes in the concentrations of atmospheric carbon dioxide (CO_2), methane (CH_4), and nitrous oxide (N_2O)—major greenhouse gases associated with anthropogenic activities—were specified. It was suggested that these gases were well mixed and their concentrations were independent of spatial coordinates. Additionally, the temporal changes observed in the longitudinal–latitudinal content of sulfate aerosol, meridional distribution of the optical thickness of volcanic aerosol, and solar constant were specified. All these data are available at http://www-pcmdi.llnl.gov/ipcc.climate_forcing.php. The duration of the given experiment was 130 years (1871–2000). A control experiment in which the contents of all atmospheric constituents did not vary in time and corresponded to the conditions of 1871 was also conducted.

(2) Experiments on modeling the climate of the 21st and 22nd centuries. The contents of carbon dioxide, methane, nitrous oxide, and sulfate aerosol in the 21st century corresponded to scenarios A1B, A2, and B1 proposed in [17]. The solar constant and the content of volcanic aerosols were specified constant and equal to their values observed in 2000. During the 22nd century, the contents of all gas constituents corresponded to 2100. The duration of each of the experiments was 200 years. The temporal behaviors of all external forcings used in [16] and of the contents of carbon dioxide, methane, nitrous oxide, and sulfate aerosol in the 21st century according to different scenarios are shown in Fig. 6. Some of the results of calculations from [16] are presented in Figs. 7–9.

From comparison of the temporal behavior of the difference between the globally averaged temperatures in the experiment on the reproduction of the climate of the 20th century and in the control experiment

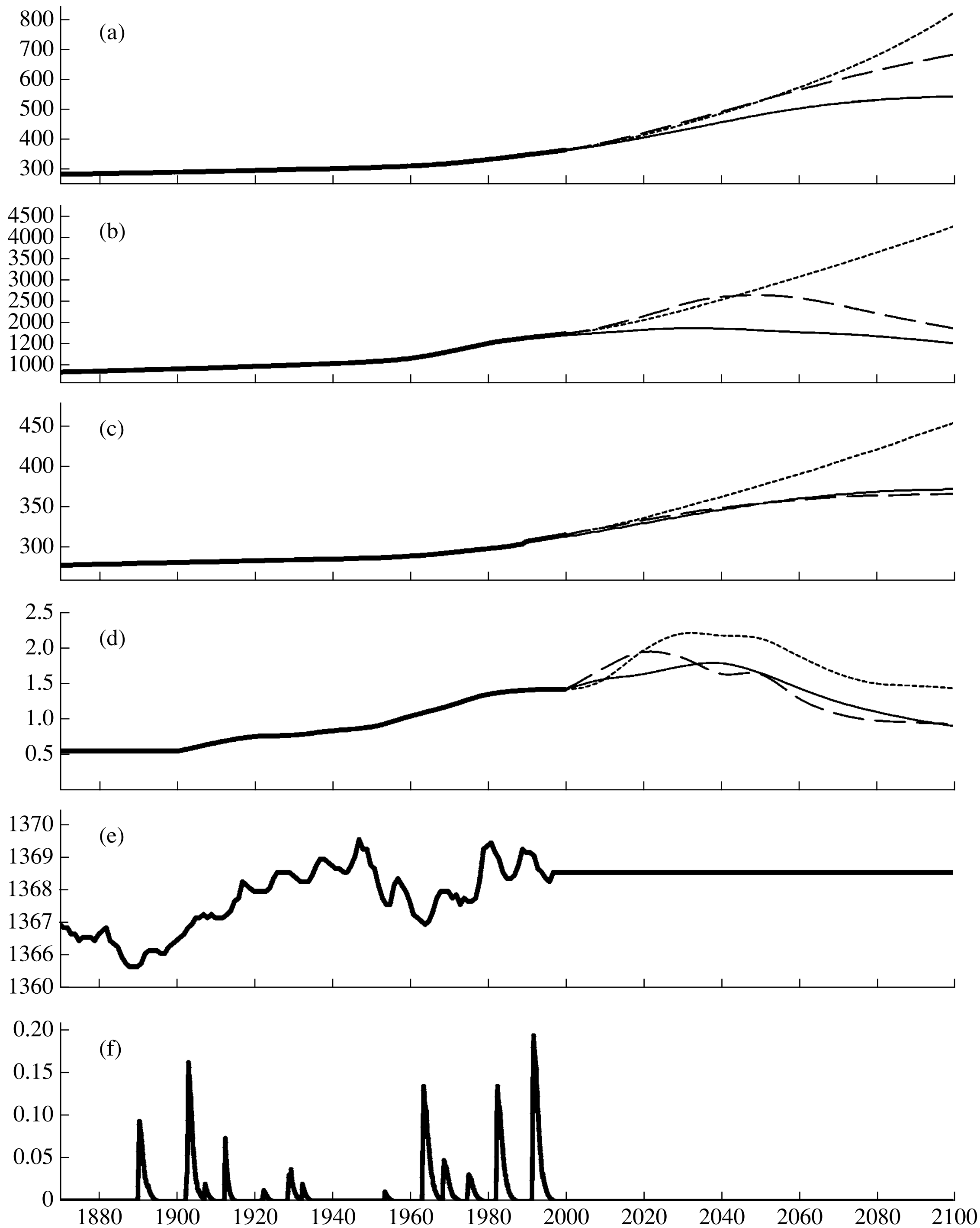

Fig. 6. Variations in the contents of (a) carbon dioxide (ppm), (b) methane (ppb), (c) nitrous oxide (ppb), and (d) integral sulfate aerosol (mg/m^2); in (e) the solar constant (W/m^2); and in (f) the integral optical thickness of volcanic aerosol (dimensionless) in (heavy solid line) the experiment for the 20th century and in the experiments with scenarios (thin solid line) B1, (dashed line) A1B, and (dotted line) A2.

with the estimate of temperature variation in 1871–2000 taken from observational data (Fig. 7), it is easy to see that the model adequately reproduces such features as the warming in 1940–1950 and its slowdown in 1960–1970. It is possible that the cause of these features is due to the presence of a maximum of the solar constant and a minimum of volcanic aerosols in 1940–1950 and a minimum of solar activity and a maximum of volcanic aerosols in 1960–1970 (Fig. 6). At the same time, it must not be ruled out that, as fol-

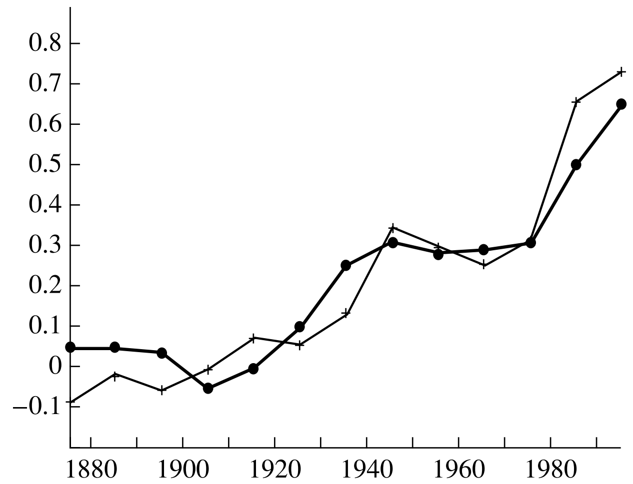

Fig. 7. (Heavy line) Variations in the globally averaged surface air temperature in 1871–2000 from observational data and (thin line) the difference constructed from the results of reproducing the climate of the 20th century and the control experiment. The data are averaged over decades.

lows from the behavior of temperature in the control experiment (Fig. 8), these features may be associated with the natural variability of the climate system.

Figure 8 presents the temporal behaviors of the globally averaged surface air temperature in the control experiment and in the experiments modeling the climate of the 20th–22nd centuries. In the experiment modeling the climate of the 20th century, a marked warming relative to the warming in the control experiment is observed as early as by the middle of the century. By the end of the century, the temperature increase reaches 0.7–0.8°, which is close to the observed warming, about 0.6–0.7°. The data of the control experiment do not contain a time interval within which the warming would is so substantial. This finding implies that the warming observed in the 20th century is most likely due to external forcings rather than to the internal variability of the atmosphere–ocean system itself. An analogous conclusion is also made from the results of other models whose results are used in [17]. According to model results, during the 21st century, a temperature increase of about 0.6 K is expected to ocuur ownig to thermal inertia of the ocean even if all forcings are fixed at the level of 2000. According to the data of the model, the increases in temperature in experiments B1, A1B, and A2 in comparison with 2000 are more clearly defined and reach 2, 3, and 5 K, respectively, by the end of the 22nd century.

The temperature changes nonuniformly over the surface during global warming. As is shown in [16], the warming is maximum in the Arctic and reaches 10 K there. In the territory of Russia, the temperature increase is 5–7 K. In the remaining part of the continents, the temperature increases by 3–5 K. The warming is the least pronounced over the tropical oceans and the Southern Ocean (2–3 K). Such a distribution of warming over the globe is characteristic of most models and is close to the results obtained via averaging the data of all CMIP models. The calculated precipitation change during warming in the INM RAS model is also typical of most models. Precipitation increases by 20 to 40% in the middle and high latitudes of both hemispheres and over the tropical Pacific. Over the most part of subtropics and the tropical Atlantic, precipitation decreases.

The maximum of warming in the Arctic is due to a considerable thawing of sea ice at the end of the summer season. Figure 9 depicts the area of sea ice in the Northern Hemisphere in March and September for the control experiment and experiments B1, A1B, and A2 on the reproduction of the climate of the 20th century. In March, the area of ice in the control experiment remains almost constant; only high-frequency oscillations occur and a small negative trend is recorded.

In experiments B1, A1B, and A2, the area of ice decreases in March and, by the end of the 22nd century, the decrease reaches 20, 30, and 50%, respectively. In September, changes in the area of sea ice are pronounced still more clearly. By the end of the 20th century, the area of ice decreases by 20 to 25% relative to the control experiment. In the 22nd century, according to the results of experiment A2, there is no ice in the Arctic; according to the results of experiment A1B, ice remains by September only in some years; and according to the results of experiment B1, the area of ice is as small as 10 to 20% of the area in the control experiment. It should be noted that, according to observational data, at the end of the 20th century, the area of Arctic sea ice in July–September turned out to be 20 to 25% smaller than in the middle of the century, whereas, this characteristic in January–March remained virtually invariant over the past 50 years. This finding corresponds to the model results presented in Fig. 9.

CONCLUSIONS

In summary, we emphasize once more that the strategy of studies (within the framework of the national climate program) of modeling climate and its global changes should be based on the following four main propositions: (i) construction of an original climate model, (ii) model implementation on parallel-computing systems, (iii) development of the mathematical theory of climate, and (iv) study of regional problems of climatic variability that are important in Russia. The experience accumulated in such studies at the Institute of Numerical Mathematics of the Russian Academy of Sciences makes it possible to state that at present there are theoretical and technological backgrounds for solving the problems related to the predic-

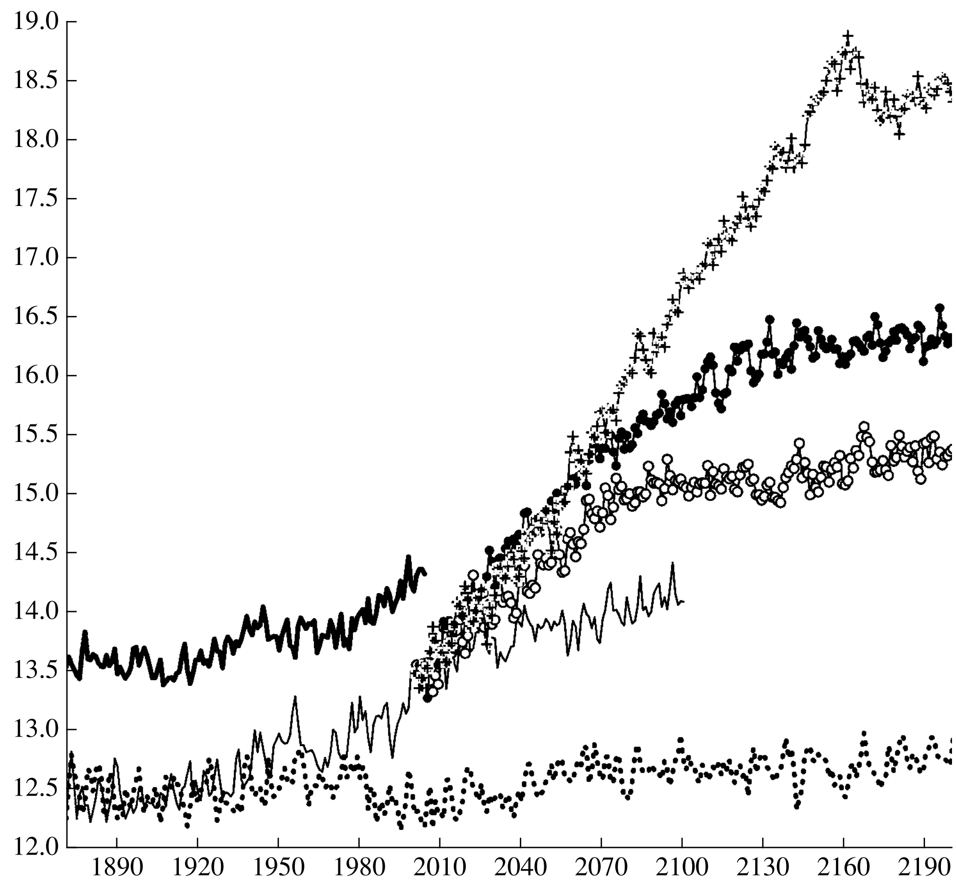

Fig. 8. Variations in the integral temperature of surface air (°C) in (dotted heavy line) the control experiment, (open circles) experiment B1, (closed circles) experiment A1B, and (crosses) experiment A2. The solid heavy line shows the observed temperature variation.

tion of climate changes, both natural and caused by human activities. The atmosphere–ocean general circulation models developed at the institute have achieved the world level of both complexity in describing physical processes and adequacy in reproducing present-day climate characteristics. The results of modeling the combined atmosphere–ocean circulation testify that further improvement of the INM RAS climate model aimed at the study of climate changes on different scales is promising.

The main directions in which the development of the mathematical theory of climate and the improvement of modeling of climate and climate change will be possible in the coming years can be formulated as follows.

(1) Mathematical theory of climate: (a) elaboration of stability theory for the attractors of climate models, (b) study of the structure of the attractors of climate models, (c) development of sensitivity theory for climate models (theorems on the linear approximation for different moments, numerical study of the linear theory of response to small perturbations, optimal perturbations, and algorithms for constructing the response operator), and (d) control theory for dissipative systems (climate control).

(2) Climate models: (a) development of parametrizations for physical processes (stochastic parametrizations), (b) improvement of coupled atmosphere–ocean models, (c) development of regional climate models and methods to assess the consequences of climate changes for the natural medium, and (d) elaboration of models of the middle and upper atmosphere for solving the problems related to "space weather."

(3) Numerical methods and parallel computations: (a) development of the theory of approximation of hydrothermodynamic equations on attractors (approximation of an attractor as a set and approximation of the measure on it), (b) approximation of the dynamics of the climate system on attractors, (c) elaboration of schemes with a specified symmetry group, (d) construction and use of spatiotemporal adaptive grids, and (e) design of computing technologies oriented toward massively parallel computing systems.

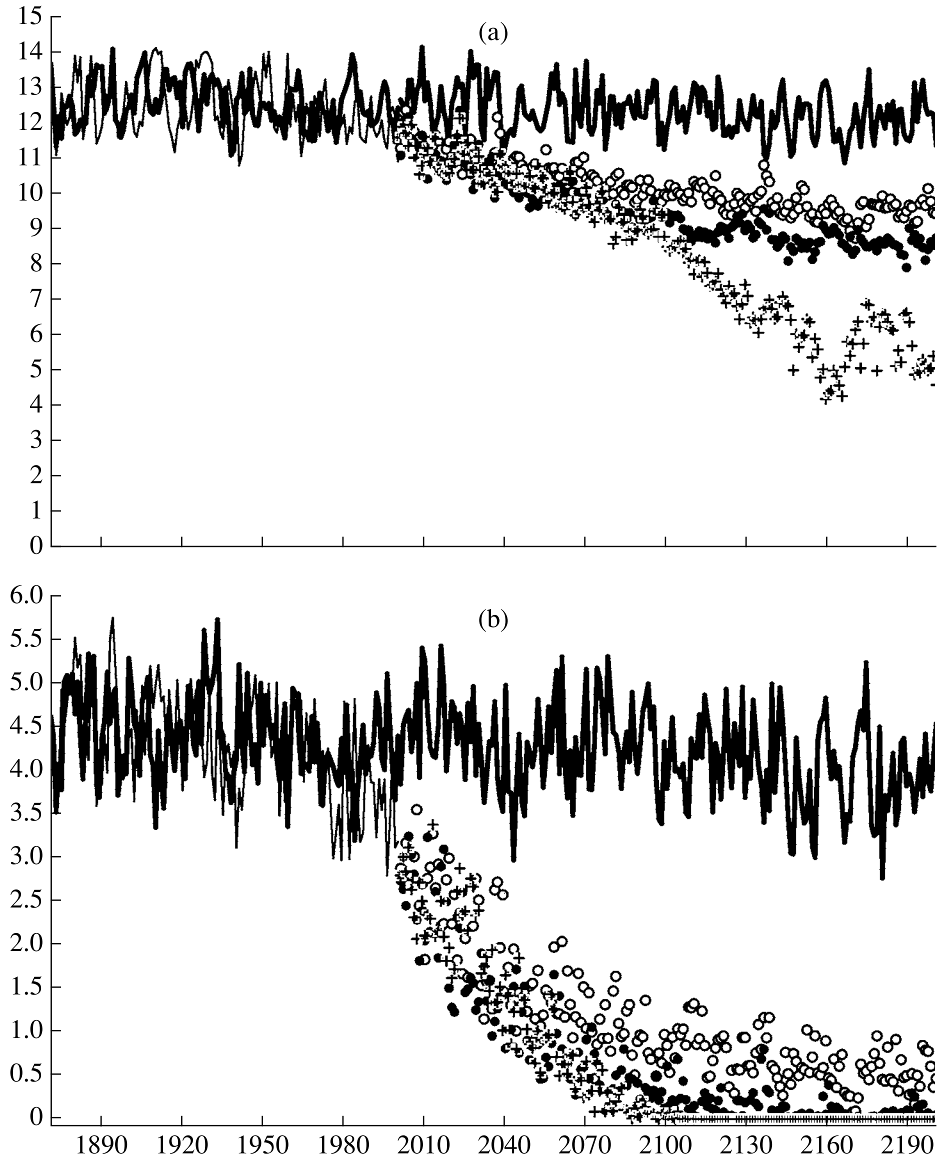

Fig. 9. Area of sea ice in the Northern Hemisphere, (10^6 km^2) in (a) March and (b) September from (heavy line) the control experiment, (thin line) the experiment on reproducing the climate of the 20th century, (open circles) experiment B1, (closed circles) experiment A1B, and (crosses) experiment A2.

The aforementioned makes it possible to hope for the elaboration of a national expert system used to obtain estimates and substantiated predictions of climate oscillations and changes on both a regional and a global scale.

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