Spectral Equivalence and Proper Clusters for Matrices from the Boundary Element Method

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ABSTRACT

The Galerkin matrices A_n from applications of the boundary element method to integral equations of the first kind usually need to be preconditioned. In the Laplace equation context, we highlight a family of preconditioners C_n that simultaneously enjoy two important properties: (a) A_n and C_n are spectrally equivalent, and (b) the eigenvalues of $C_n^{-1}A_n$ have a proper cluster at unity. In the Helmholtz equation context, we prove the spectral equivalence for the so-called second Galerkin matrices and that the eigenvalues of $C_n^{-1}A_n$ still have a proper cluster at unity. We then show that some circulant integral approximate operator (CIAO) preconditioners belong to this family, including the well-known optimal CIAO. Consequently, if we use the preconditioned conjugate gradients to solve the problems, the number of iterations for a prescribed accuracy does not depend on n, and, what is more, the convergence rate is superlinear.

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