# Tensor-Based Algorithms

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1. Pure matrices and pure tensors. Skeleton decomposition and how it becomes the Singular Value Decomposition. Low-rank approximations of matrices in the Frobenius and spectral norms. Muliplication of tensors.

2. Tensor representations and approximations by sums of pure tensors. Relation with fast matrix multiplication. Minimal pure tensor decompositions and tensor rank. Tensors with two sections. Uncloseness property.

3. Tensor-Train (TT) decompositions. TT-ranks and ranks of associated matrices. Construction of TT for a tensor given by its elements. Approximation of a given TT by another TT with smaller TT-ranks.

4. Uniqueness property of minimal pure tensor decompositions. Decompositions with linear independent vectors along each dimension.

5. Kruskal condition of uniqueness and its generalizations. Basic permutation lemma. Complete proof of uniqueness using matrices of minors.