Applications of tensor trains to numerical simulation

Sergey Matveev

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Abstract

These seminars are devoted to series of practical examples when low rank matrix and tensor decompositions allow users to obtain efficient numerical algorithms. In particular, we will discuss examples of efficient algorithms for evaluation of bi-linear and multi-linear convolutions, discuss the matrix cross interpolation algorithm and compare complexities of straight-forward algorithms with tensor-based approaches. The main practical example would be class of Smoluchowski-typed aggregation-fragmentation equations with number of possible modifications (e.g. consideration multi-particle collisions).

Plan:

- Seminar 1 (Moscow): Matrix cross interpolation algorithm, complexity of method, cross decomposition, low-rank matrix matvec operation, low-rank bi-linear convolution operation. Application of proposed algorithms to solution of aggregation equations and analysis of complexity.
- Seminar 2 (Moscow): Multi-particle aggregation equations and exponential growth of complexity of straight-forward computations. Operations in pure-tensor (CP) format – their complexity, problem with finding the decomposition. The same operations for right-hand side in tensor train (TT) format – higher complexity but existence of algorithms constructing the decomposition for kernel coefficients. Small examples for low-rank TT-functions
- Seminar 3 (Rome): Basic operations in TT format: elementwise sum, product, scalar product. Complexities of operations in TT format. Some ideas about TT-cross algorithm and its "cost" (without strict algorithm). "Naive" TT-cross based algorithm for multicomponent Smoluchowski coagulation equation.
- Seminar 4 (Rome): More clever algorithm for multi-component Smoluchowski equation, analysis of its complexity. Examples of low-rank kernel coefficients.

Materials:

References

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[3] Matveev, S. A., Zheltkov, D. A., Tyrtyshnikov, E. E., and Smirnov, A. P. (2016). Tensor train versus Monte Carlo for the multicomponent Smoluchowski coagulation equation. Journal of Computational Physics, 316, 164-179.

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[5] Oseledets, Ivan, and Eugene Tyrtyshnikov. "TT-cross approximation for multidimensional arrays." Linear Algebra and its Applications 432.1 (2010): 70-88.